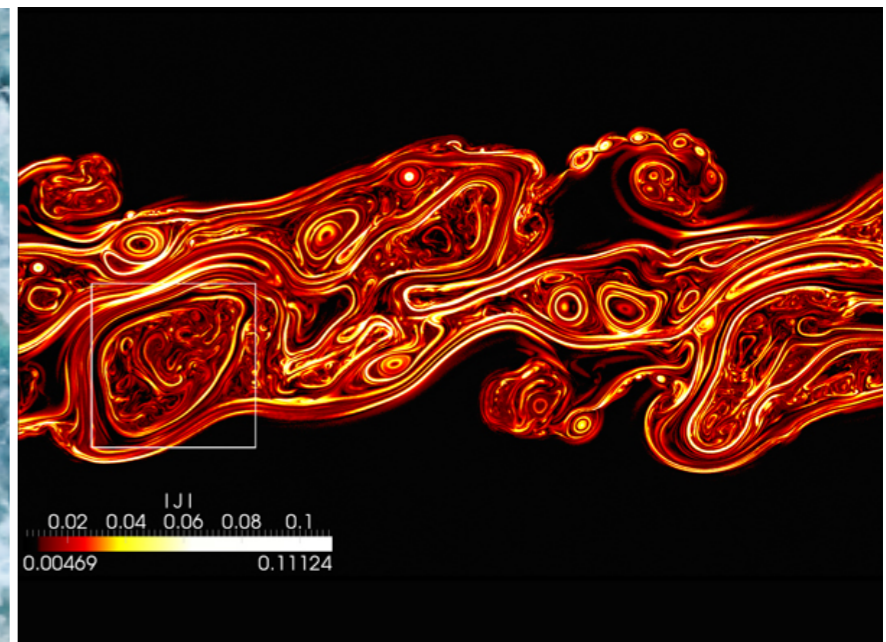
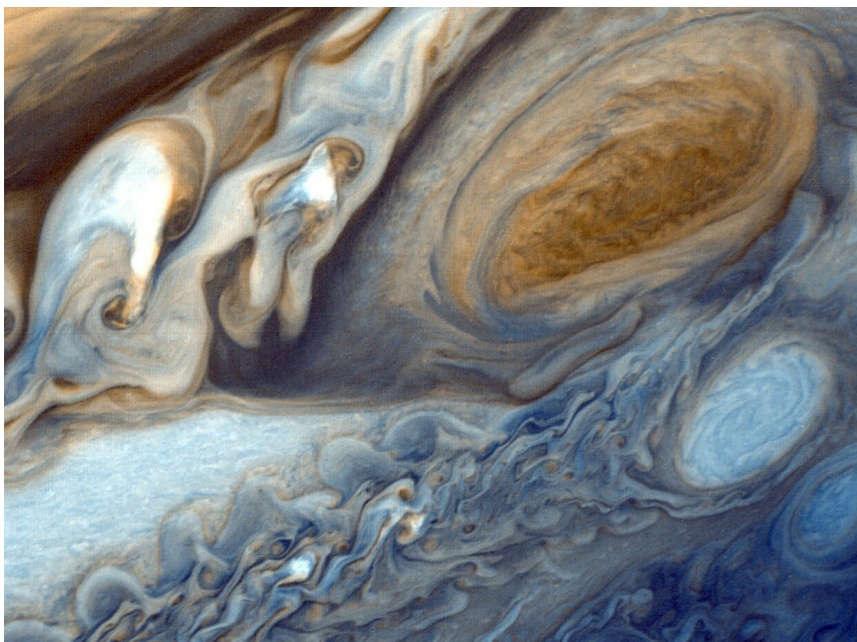


# Plasma Turbulence

Natasha Jeffrey, Northumbria University





- **Part 1: Basics of turbulence**

- *What is turbulence?*
- *Properties of turbulent flows.*
- *Fluid versus MHD plasma turbulence.*

- **Part 2: Turbulence in solar and space plasmas**

- *Role of turbulence in solar and space plasmas.*
- *Examples of turbulence in solar plasma.*
- *Examples of turbulence in space plasma.*

- **Summary and questions**



# What is turbulence?

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- What words do you associate with turbulence???

# What is turbulence?

- What words do you associate with turbulence???

“disturbance”

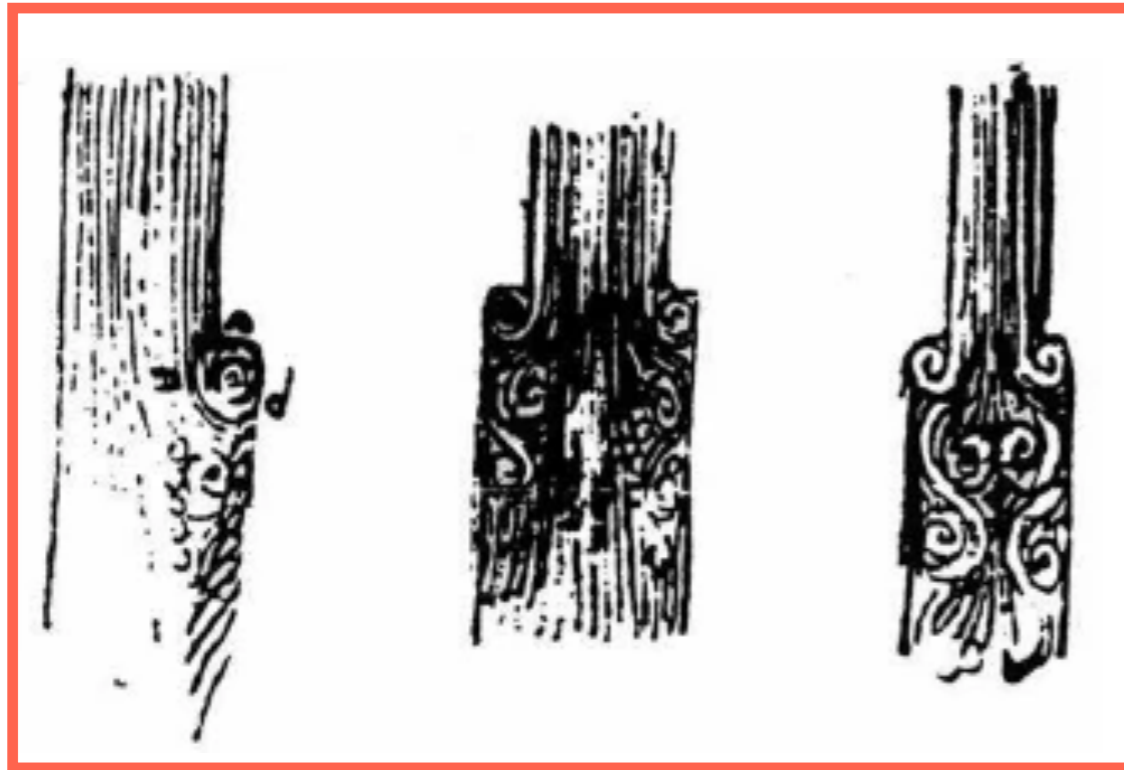
“disruption”

“chaotic”

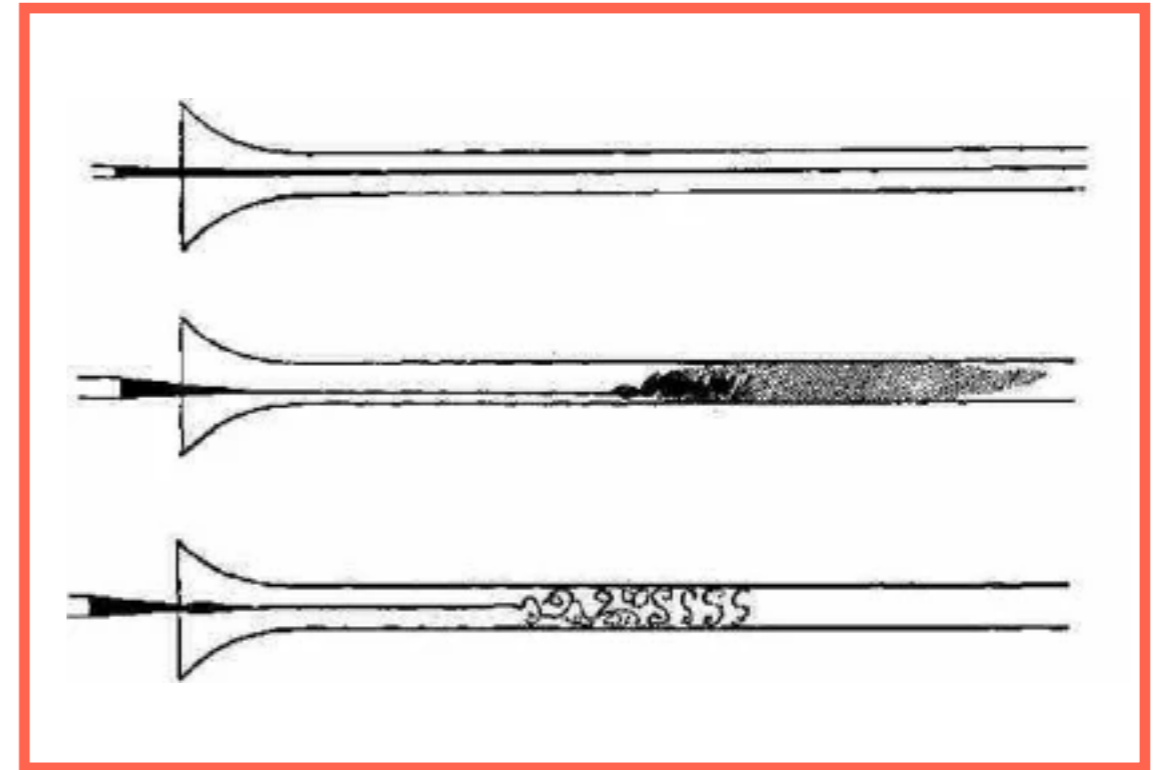
“irregular”

“unsettled”

*Latin: turba means something confusing or something which does not follow an ordered plan.*



Studies of turbulence by Leonardo  
Da Vinci (Frisch 1995)



Original pictures by Reynolds 1883  
(Bruno & Carbone 2013).

# What is turbulence?

- What words do you associate with turbulence???

*“disturbance”*

*“disruption”*

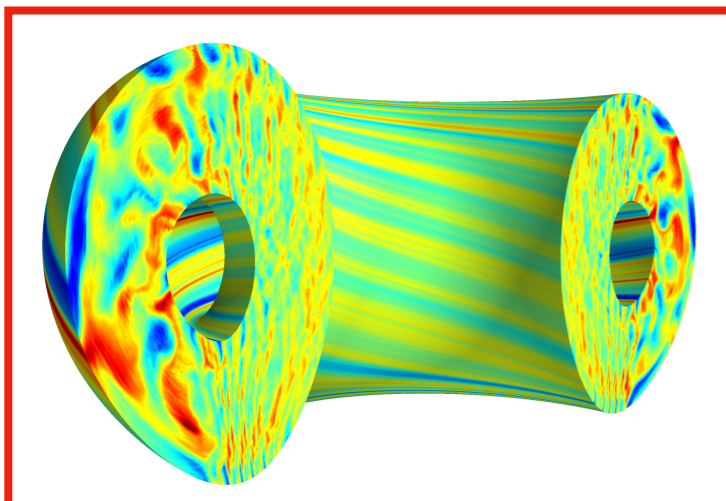
*“chaotic”*

*“irregular”*

*“unsettled”*

*Latin: turba means something confusing or something which does not follow an ordered plan.*

- Understanding turbulence is important as it appears everywhere in nature...



# What is turbulence?

- “While fluids can flow in a variety of ways, turbulence describes the extremely complicated, chaotic motion that fluids frequently exhibit in natural and engineered systems.”
- Fluid flow can be described as **laminar** or **turbulent**:
  - **laminar flow** occurs at low Reynolds numbers, where viscous forces are dominant (leading to smooth, constant motion).
  - **turbulent flow** occurs at high Reynolds numbers, where inertial forces are dominant (leading to chaotic eddies, vortices and other flow instabilities).

$$Re = \frac{\rho v L}{\mu}$$

$\rho$  is the density of the fluid

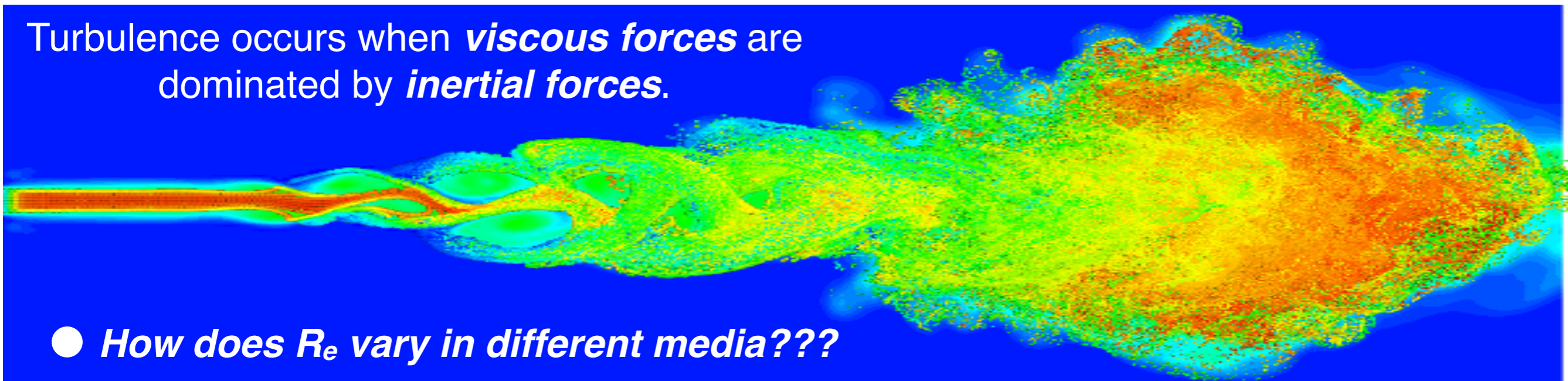
$v$  is a characteristic velocity of the fluid with respect to the object

$L$  is a characteristic linear dimension

$\mu$  is the dynamic viscosity of the fluid

Turbulence occurs when *viscous forces* are dominated by *inertial forces*.

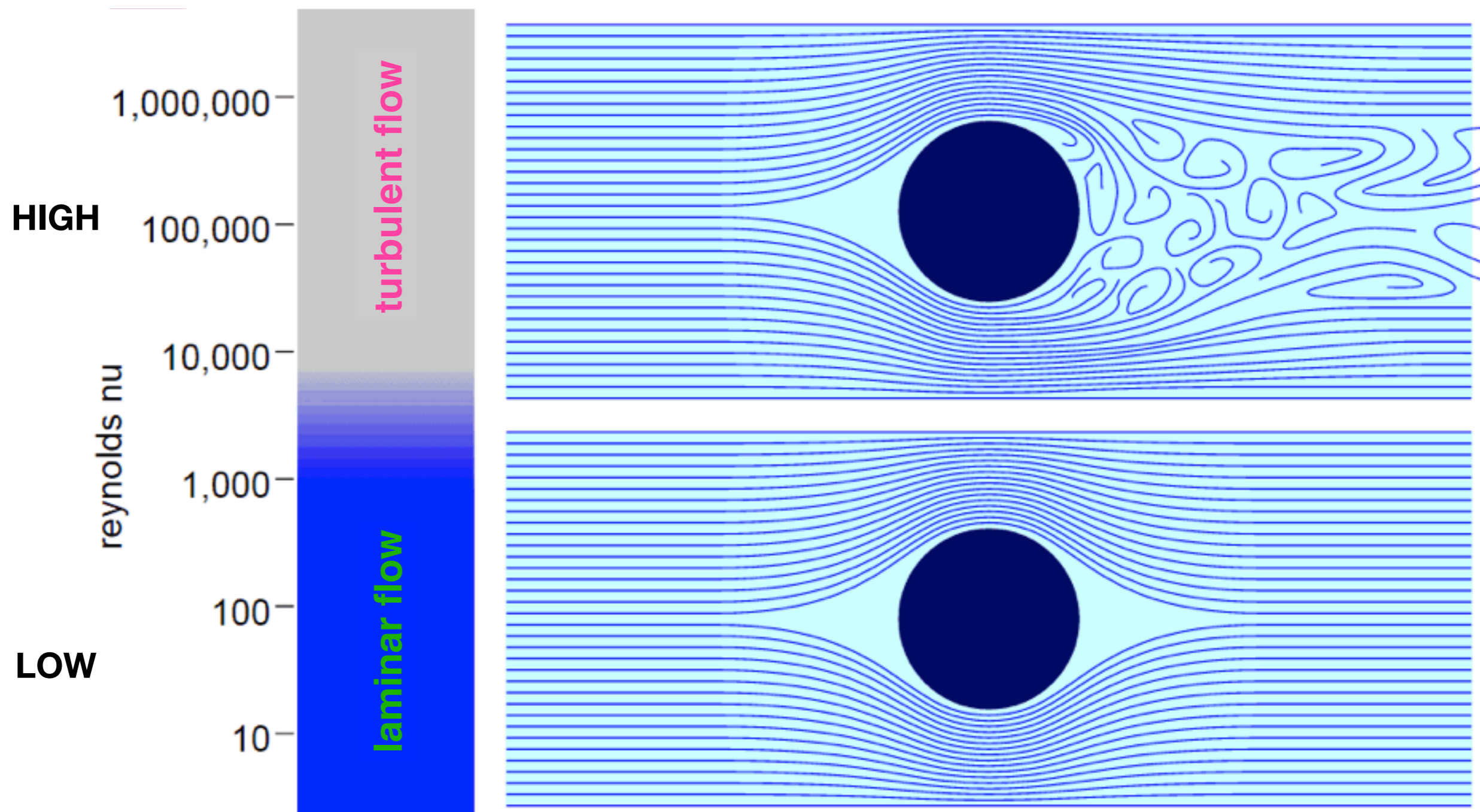
- How does  $Re$  vary in different media???





# What is turbulence?

- *“While fluids can flow in a variety of ways, turbulence describes the extremely complicated, chaotic motion that fluids frequently exhibit in natural and engineered systems.”*
- Fluid flow can be described as **laminar** or **turbulent**:

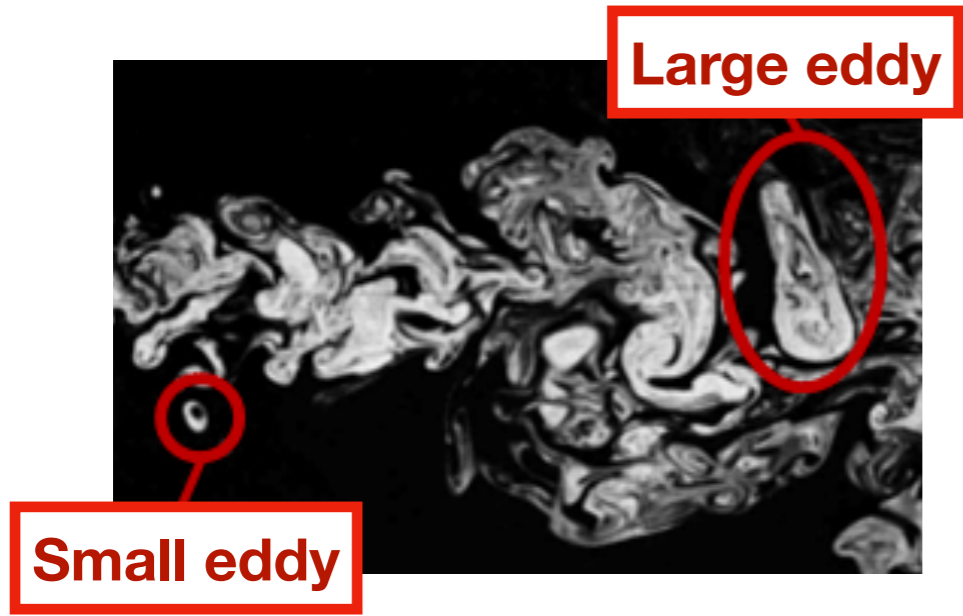


- **Eddy** - “rotating fluid element” or a “turbulent motion” localised over a region of size  $l$  and  $\sim$  coherent over this region.
- **Characteristic quantity** - the length / timescale / velocity etc. over which a process takes place.
- **Turnover time** - the timescale over which an eddy of scale  $l$  undergoes significant distortion.
- **Correlation length** - defines a length scale over which the deviations from the average stop being similar / measure of the order of the system / relationship between neighbouring fluctuations.
- **Energy power spectrum** - usually a plot of energy  $E(k)$  versus wavenumber  $k$  telling us how the energy of the system is distributed amongst the different scales.
- **Energy cascade** - defines the flow of energy between different length scales from injection to dissipation, via the inertial scale.
- **Inner and outer scales** - Scales related to the lengths at which injection and dissipation occur. A Kolmogorov assumption occurs over the inertial range i.e.,  $1/L \ll k \ll 1/l$ .
- **Incompressible** - fluid density is the same everywhere.
- **Viscosity** - measure of a fluid’s resistance to flow.
- **Homogeneous** - same properties at every point in space / all points in the fluid.
- **Isotropic / Anisotropic** - no directional preference / directional preference.

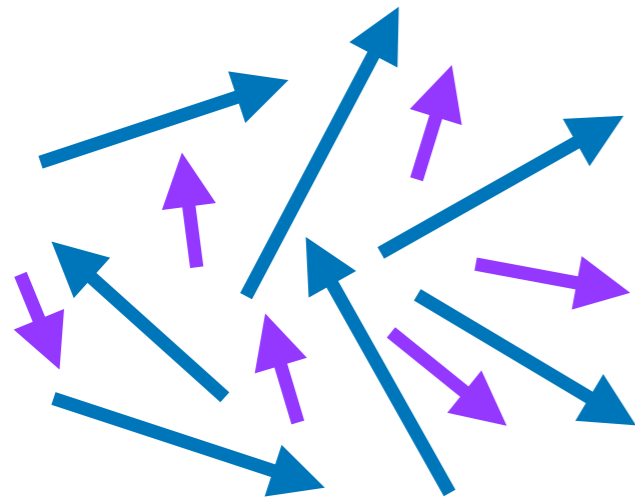
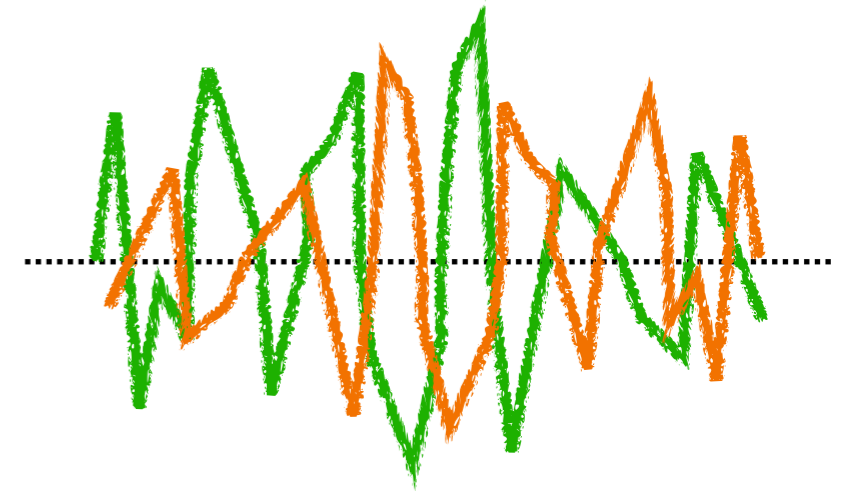


# Properties of fluid turbulence

● **Main properties of fluid turbulence:**

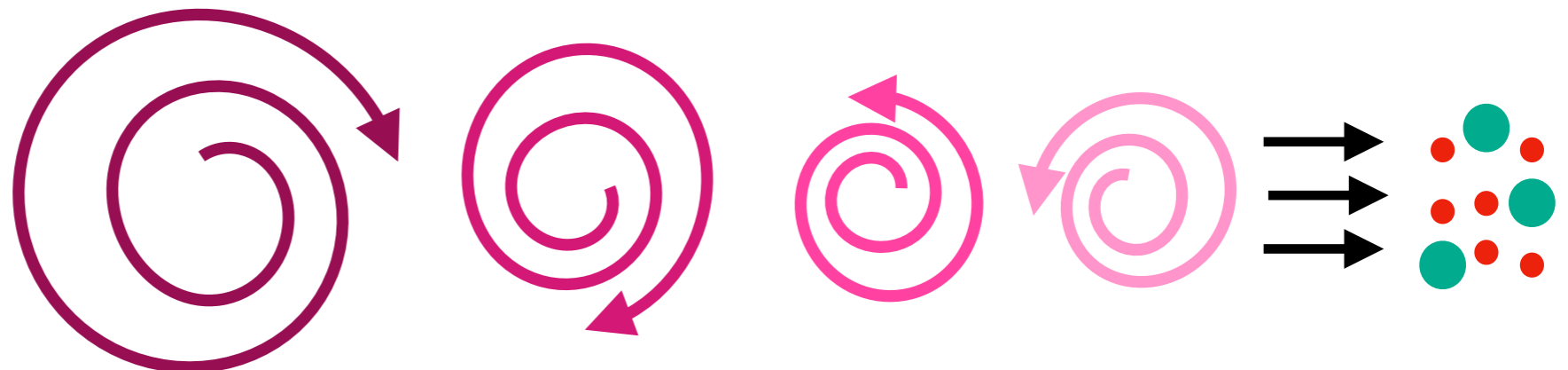


- Randomly fluctuating parameters ( $v$ ,  $P$ ,  $T$ ) with erratic motions and different size eddies.
- Irregularity (space, time).
- High Reynolds number.



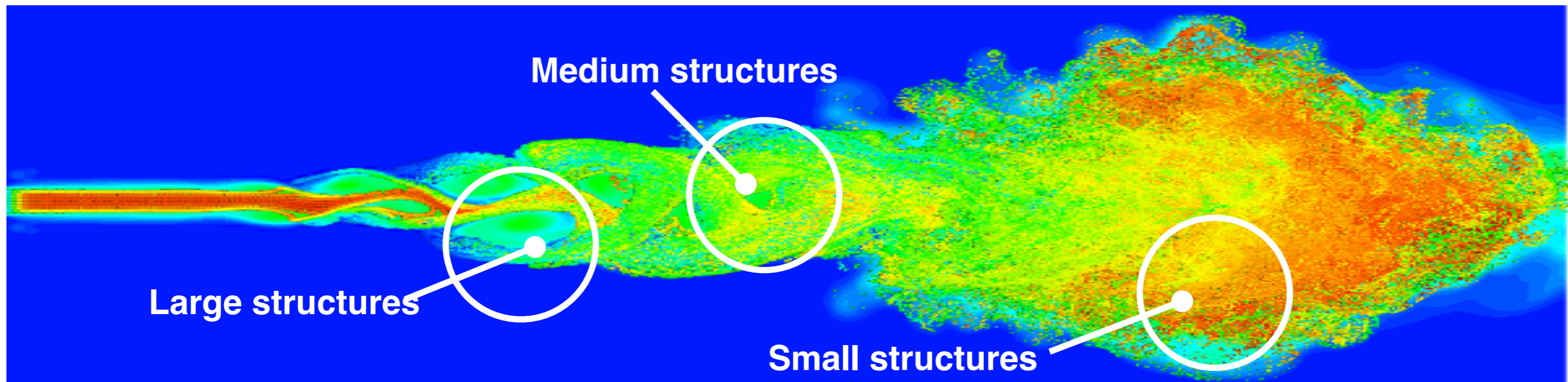
- Flow pattern is random (or "stochastic") with no preferred direction.
- Diffusive nature.

- Dissipative.

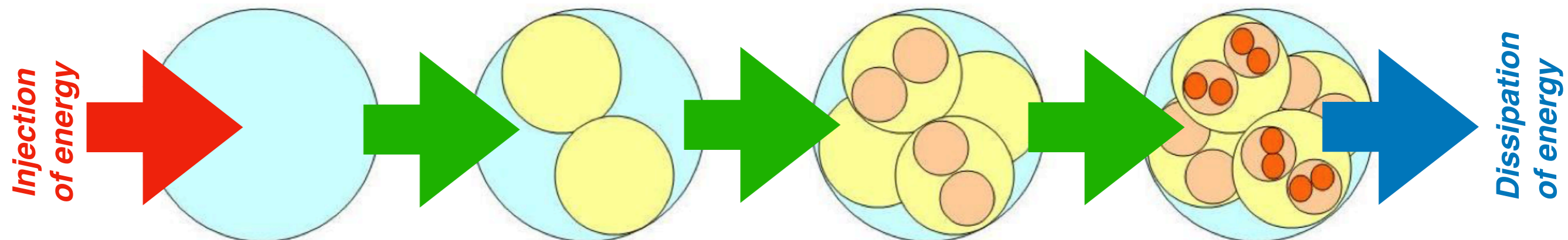


# Turbulent energy cascade

- Turbulent motions occur over a wide range of length and time scales.
- Energy is **input at large scales** and energy is **dissipated at small scales**.
- There are three main intervals in the energy cascade: (1) **Generation of energy**, (2) **Inertial range (steady state)**, (3) **Dissipation**.

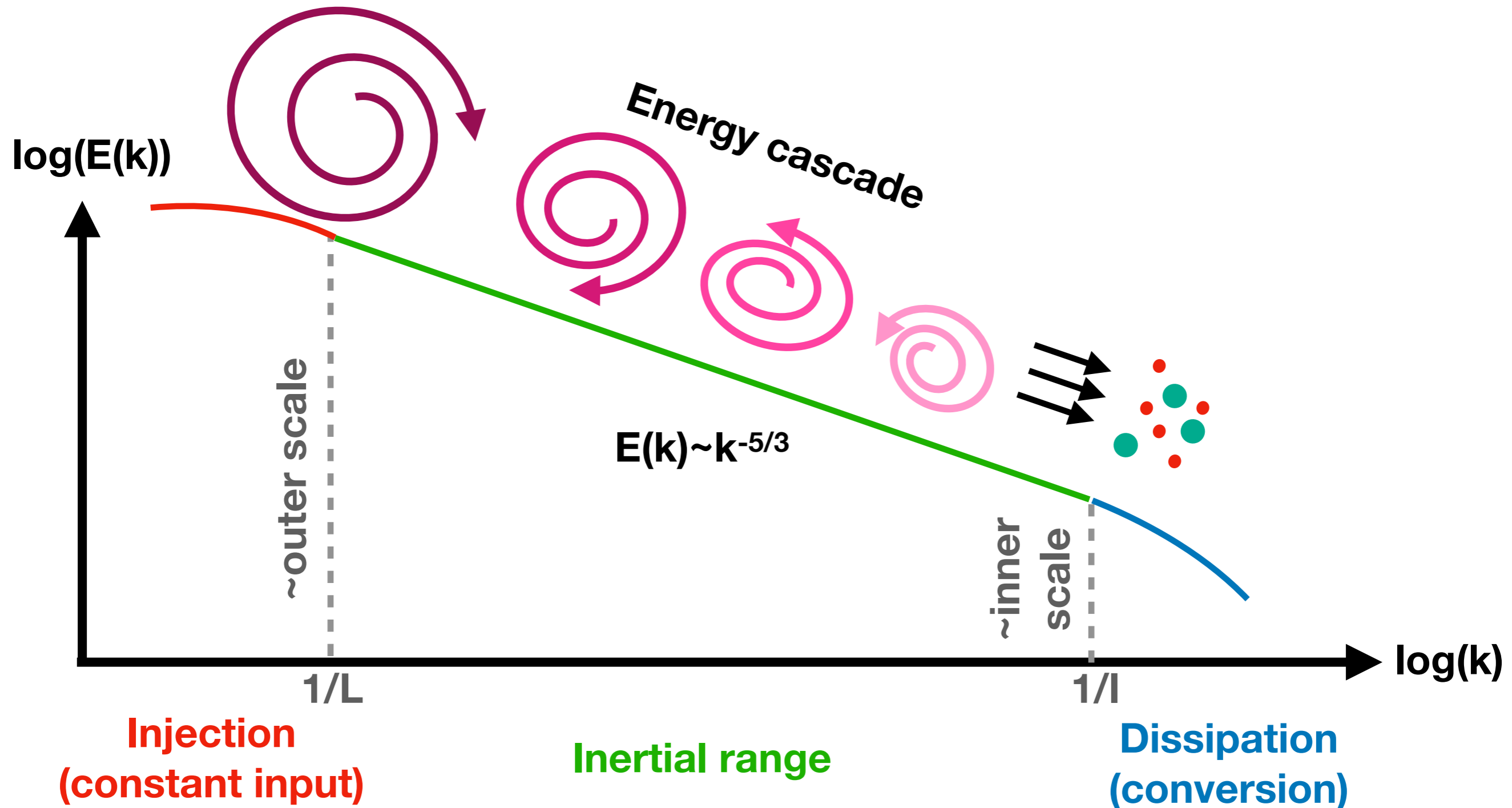


- When energy is supplied at large scales, it gets redistributed over fluctuations of different scales, and removed at small scales where viscosity can finally dominate.



# Turbulent energy cascade

- **Hydrodynamic turbulence is universal**, the energy cascade in the inertial range does not depend on the energy-injection mechanism or on the dissipation mechanism.



- The slope of  $-5/3$  in the inertial range is known as the **Kolmogorov spectrum**.

# Kolmogorov spectrum

- **Assumptions:** turbulence is incompressible, homogeneous and isotropic.
- **Assumptions:** energy transfer rate is constant from large eddies to small eddies.

Eddy turnover time,  $t$  :  $t = \frac{l}{u}$  and wavenumber,  $k$  :  $k = \frac{2\pi}{l}$  *small eddies have large  $k$*

Turbulent kinetic energy per unit mass :  $u^2 = \int_0^\infty E(k) dk$  *i.e., integral over all scales.*

Energy transfer rate / dissipation rate:  $\epsilon = \frac{\text{energy}}{\text{time}} = \frac{u^2}{t} = u^3 k = \text{constant}$

Hence,  $u = \epsilon^{1/3} k^{-1/3}$

Energy spectrum  $E(k)$  :  $E(k) = \frac{u^2}{k} = \frac{\epsilon^{2/3} k^{-2/3}}{k} = \epsilon^{2/3} k^{-5/3}$

$E(k) \sim k^{-5/3}$  is known as the **Kolmogorov spectrum**, universal amongst all inertial scales between the energy injection (outer) and dissipation (inner) scales, i.e.,

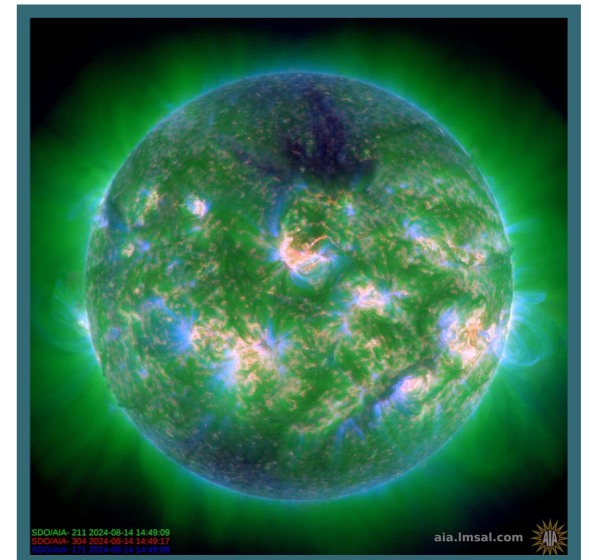
$$1/L \ll k \ll 1/l$$

- Most astrophysical plasmas are magnetised leading to MHD plasma turbulence.
- MHD turbulence describes turbulence in an electrically conducting, magnetised fluid.
- The presence of a magnetic field at larger scales introduces anisotropy.
- The energy cascade in different directions is now different (main difference to fluid turbulence with no preferred direction).
- Astrophysical sources will only be turbulent if there are suitable forces to drive the turbulence.
- We also define the **magnetic Reynolds number**:

$$R_m = \frac{UL}{\eta} \sim \frac{\text{induction}}{\text{diffusion}}$$

**U** is a characteristic velocity of the fluid  
**L** is a characteristic linear dimension  
 $\eta$  is the magnetic diffusivity

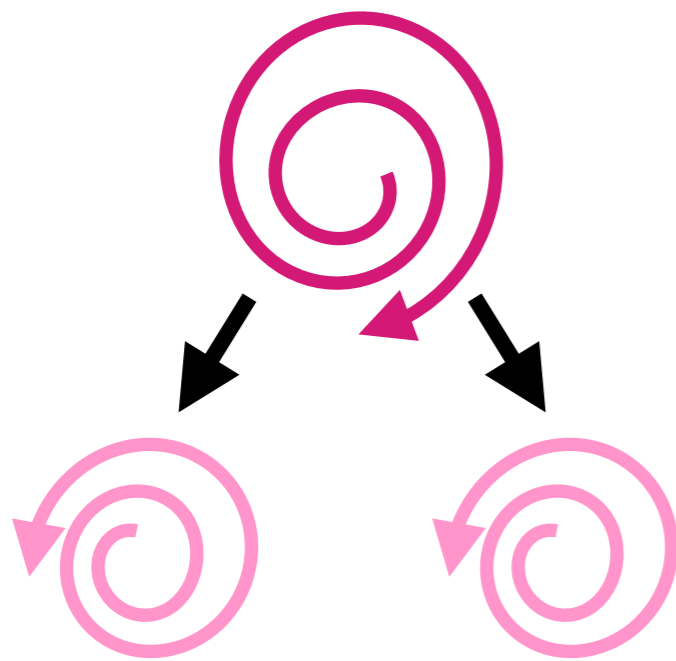
- **Magnetic diffusivity** - rate of magnetic diffusion (“magnetic viscosity”).



# MHD plasma turbulence

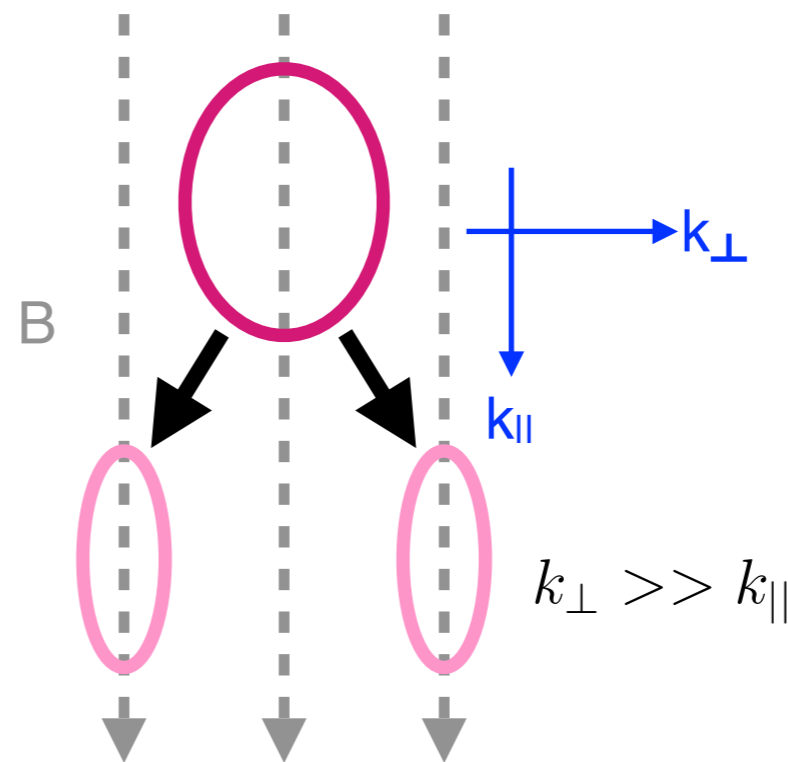
- Most astrophysical plasmas are magnetised leading to MHD plasma turbulence.
- MHD turbulence describes turbulence in an electrically conducting, magnetised fluid.
- The presence of a magnetic field at larger scales introduces anisotropy.

## Hydrodynamic turbulence

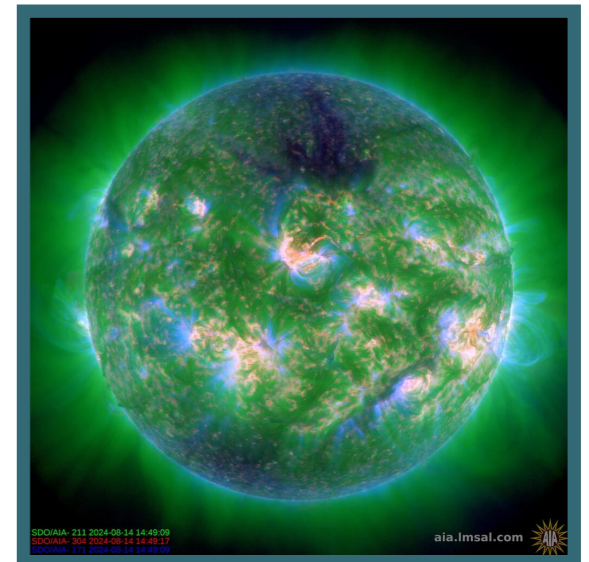


Isotropic eddies

## MHD turbulence



“Elongated eddies”



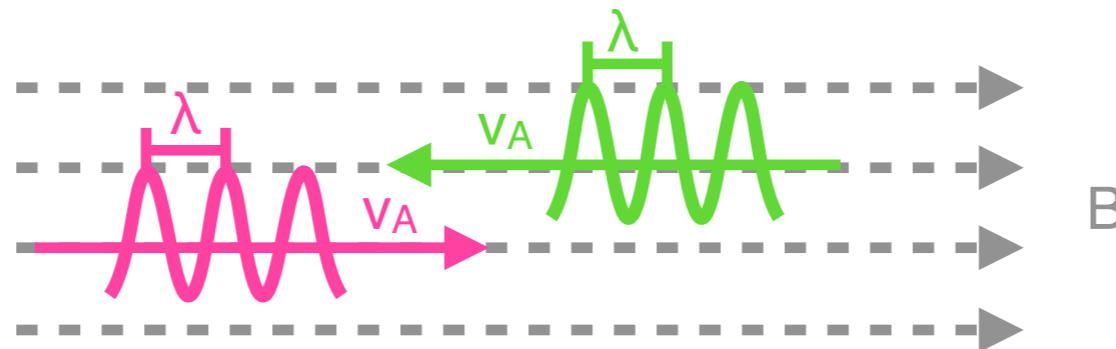
- In MHD, motion along B is free, but it is limited across B.

# Iroshnikov-Kraichnan spectrum

- Iroshnikov (1963) & Kraichnan (1965) (IK) determined the cascade spectrum in the inertial range for **weak MHD turbulence** with a **strong large-scale magnetic field**.

**We will discuss this assumption again later when we look at turbulence in the solar wind!**

- A plasma threaded by a straight uniform magnetic field  $B$  is considered. It is assumed that the **turbulence is weak**, i.e., many interactions are required to transfer energy to smaller scales.
- MHD turbulence can be created by counter-propagating magnetic disturbances such as **Alfvén waves**.



- In MHD turbulence, a new timescale becomes important and this is the **Alfvén timescale** ( $\sim$  period of an Alfvén wave).

$$t_A \approx \frac{\lambda}{v_A}$$

**The above assumptions lead to a Iroshnikov-Kraichnan spectrum (opposite wave packets interact weakly), which has a flatter spectrum than Kolmogorov (slower energy cascade).**

# Iroshnikov-Kraichnan spectrum

- Iroshnikov (1963) & Kraichnan (1965) (IK) determined the cascade spectrum in the inertial range for **weak MHD turbulence** with a **strong large-scale magnetic field**.

**We will discuss this assumption again later when we look at turbulence in the solar wind!**

- Number of interactions  $N$  required to deform the packet ( $u$  now represents components of fluctuating velocity):

$$N = \left( \frac{v_A}{u} \right)^2$$

- Dominant timescale (Alfvénic time):

$$t = N \frac{\lambda}{v_A} = \left( \frac{v_A}{u} \right)^2 \frac{\lambda}{v_A} = \frac{v_A}{ku^2}$$

- **Turbulent kinetic energy per unit mass :**

$$u^2 = \int_0^\infty E(k) dk$$

- **Energy transfer rate / dissipation rate:**

$$\epsilon = \frac{\text{energy}}{\text{time}} = \frac{u^2}{t} = u^4 k = \text{constant}$$

- Hence,

$$u = \epsilon^{1/4} k^{-1/4}$$

- **IK energy spectrum  $E(k)$ :**

$$E(k) = \frac{u^2}{k} = \frac{\epsilon^{1/2} k^{-1/2}}{k} = \epsilon^{1/2} k^{-3/2}$$



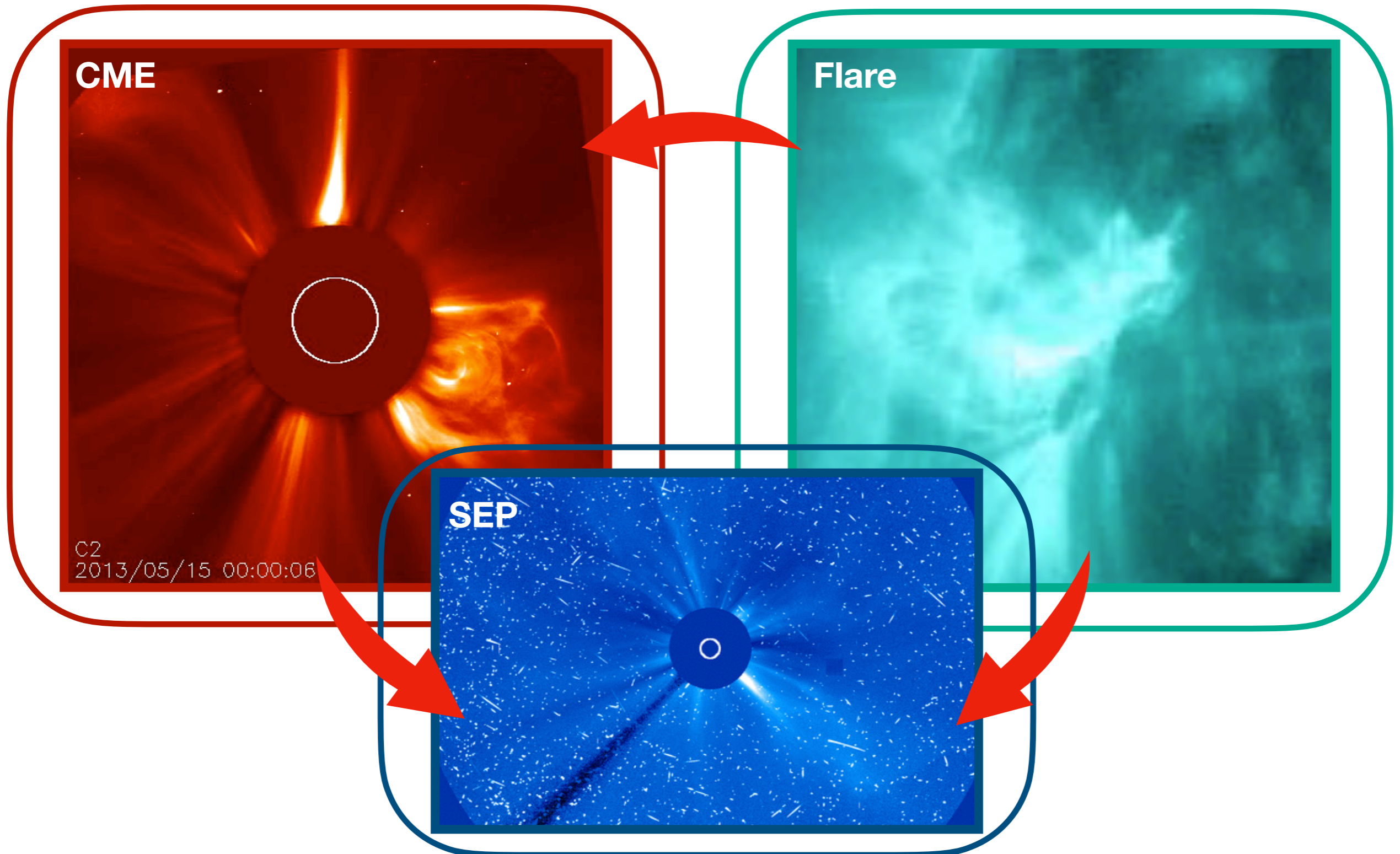
Turbulence is present and plays a key role in the behaviour and evolution of plasma in solar and space physics.

- Here, we will briefly discuss three examples:
  1. Spectral line broadening and the role of turbulence in solar flares.
  2. In-situ measurements and solar wind turbulence
  3. Radio measurements and solar wind turbulence

There are still many challenges associated with understanding MHD plasma turbulence (e.g., development, dissipation)!

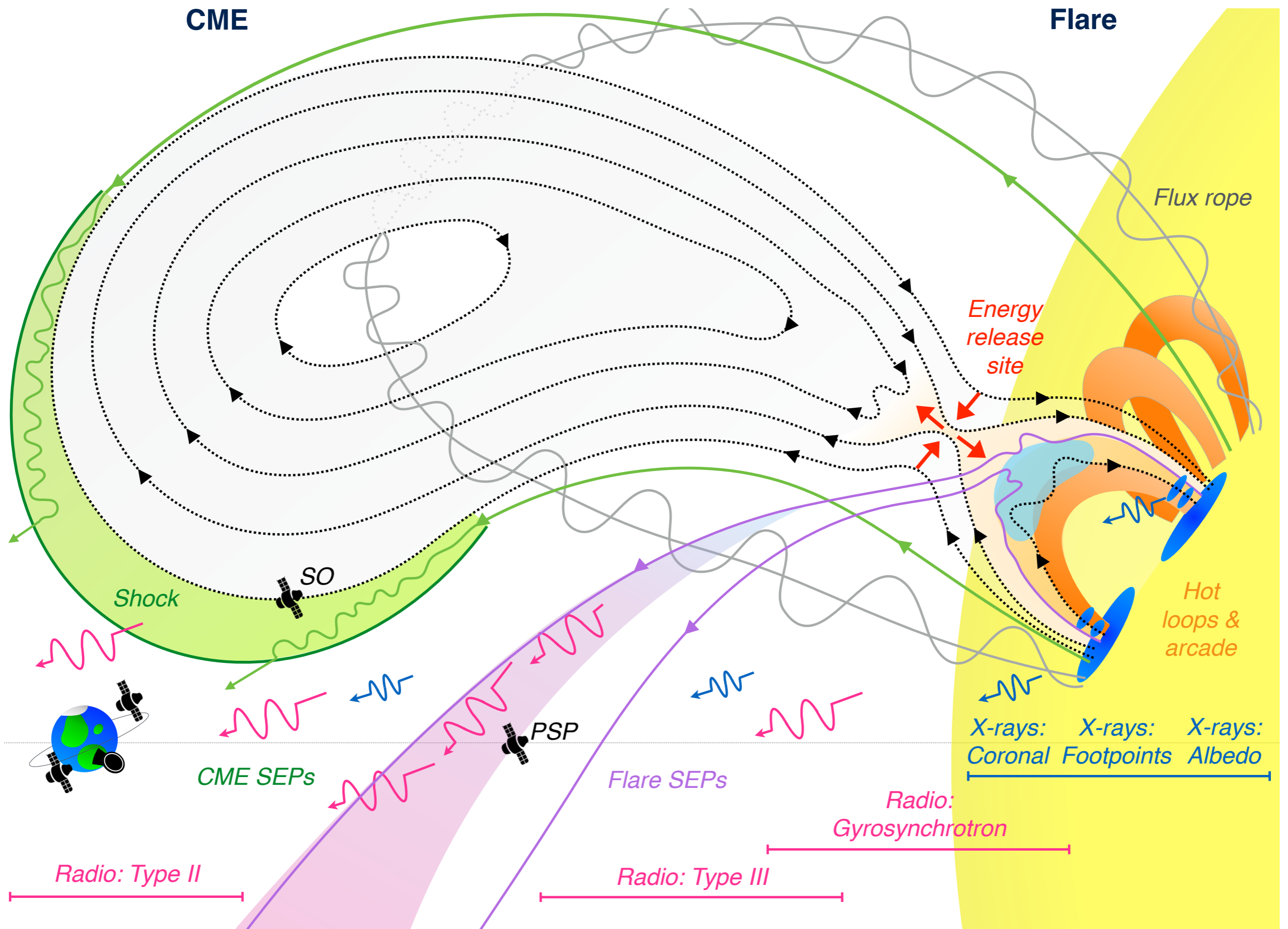
# Solar Energetic Particles

- The Sun is an efficient particle accelerator producing energetic electrons, protons and ions during transient events e.g., flares and coronal mass ejections (CMEs).





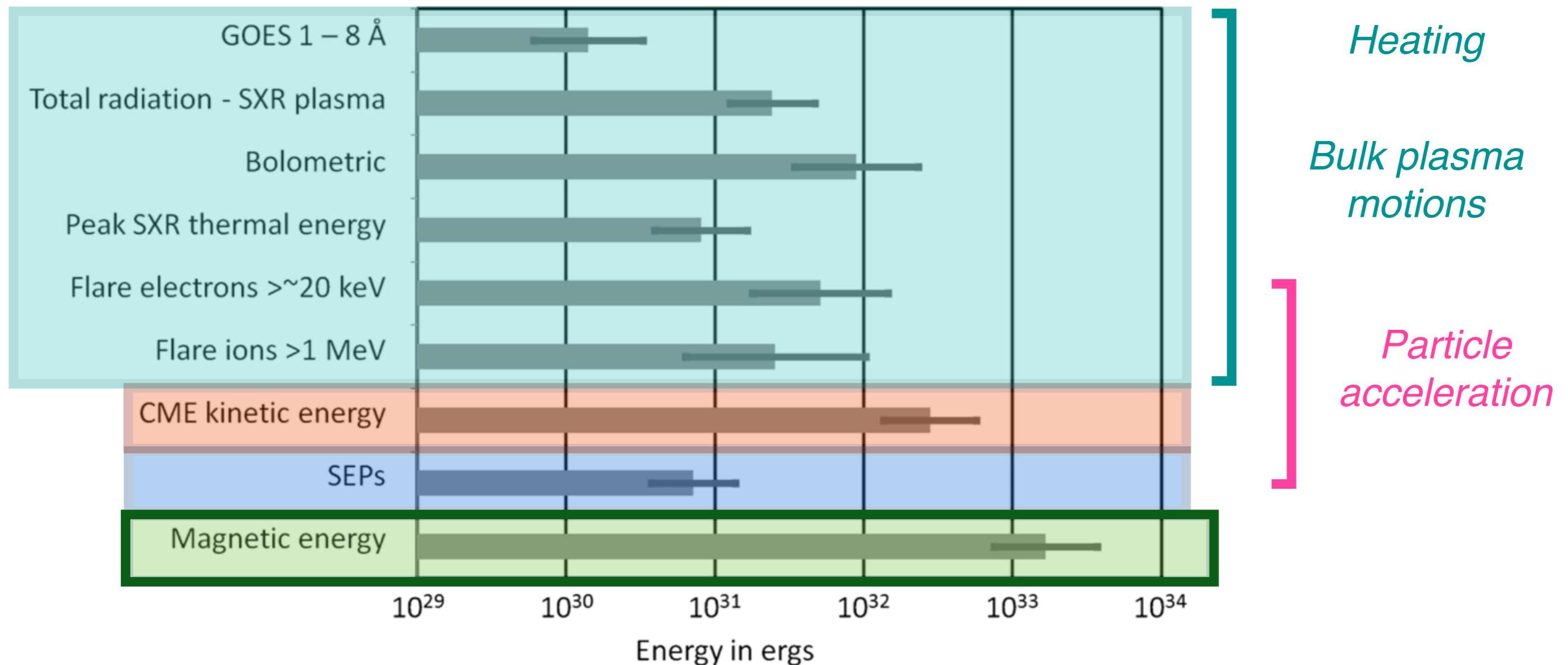
# Solar Energetic Particles



# Solar Flare Energy Partition

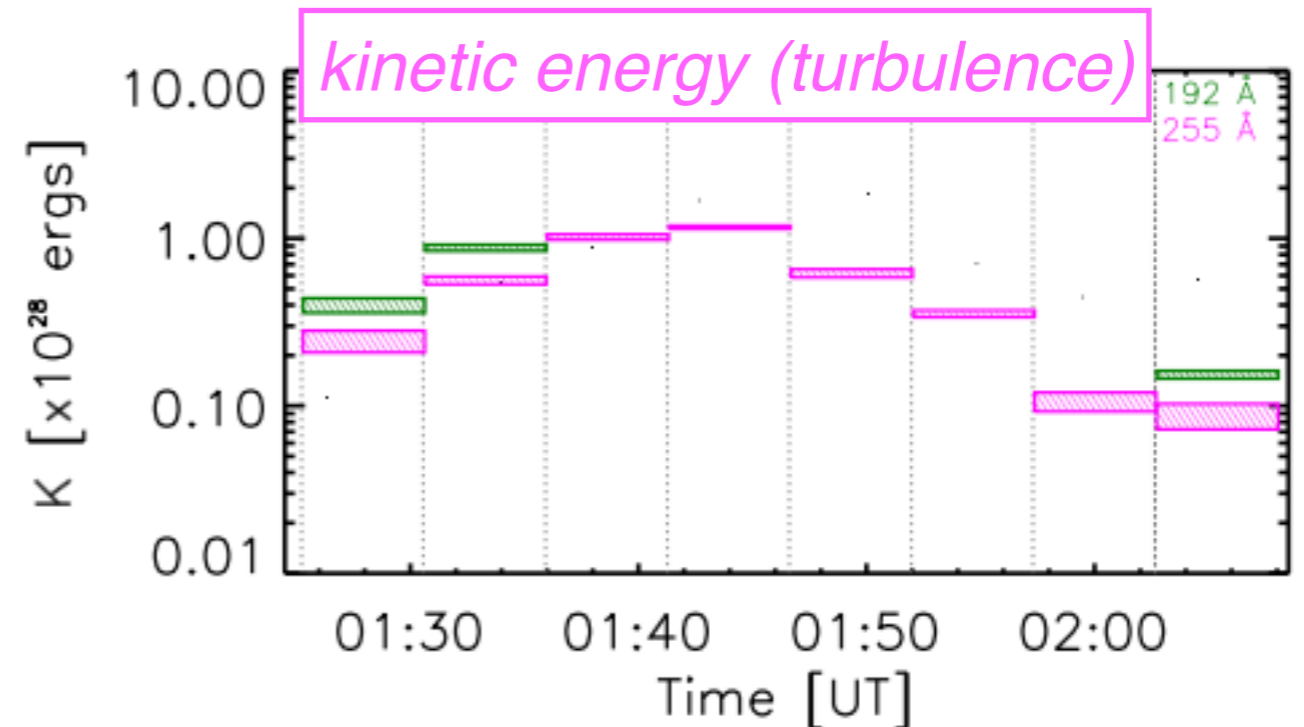
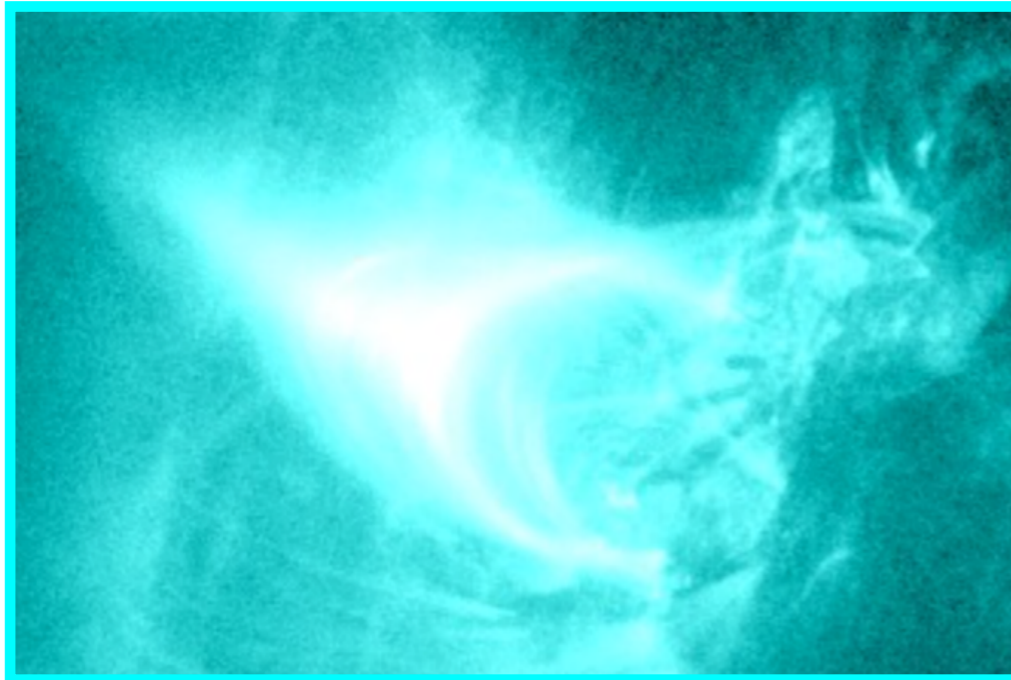
Understanding solar flares means we have to understand how energy is released, dissipated and deposited, at all stages.

- A medium solar flare releases about  $10^{32}$  erg (where 1 megaton of TNT =  $4 \times 10^{22}$  erg).
- The **magnetic energy** is transformed into different energies *e.g.* Emslie et al. 2012.



A substantial fraction of the magnetic energy goes into non-thermal electrons.

e.g., Emslie et al. 2012, Warmuth & Mann 2016, Aschwanden et al. 2017.

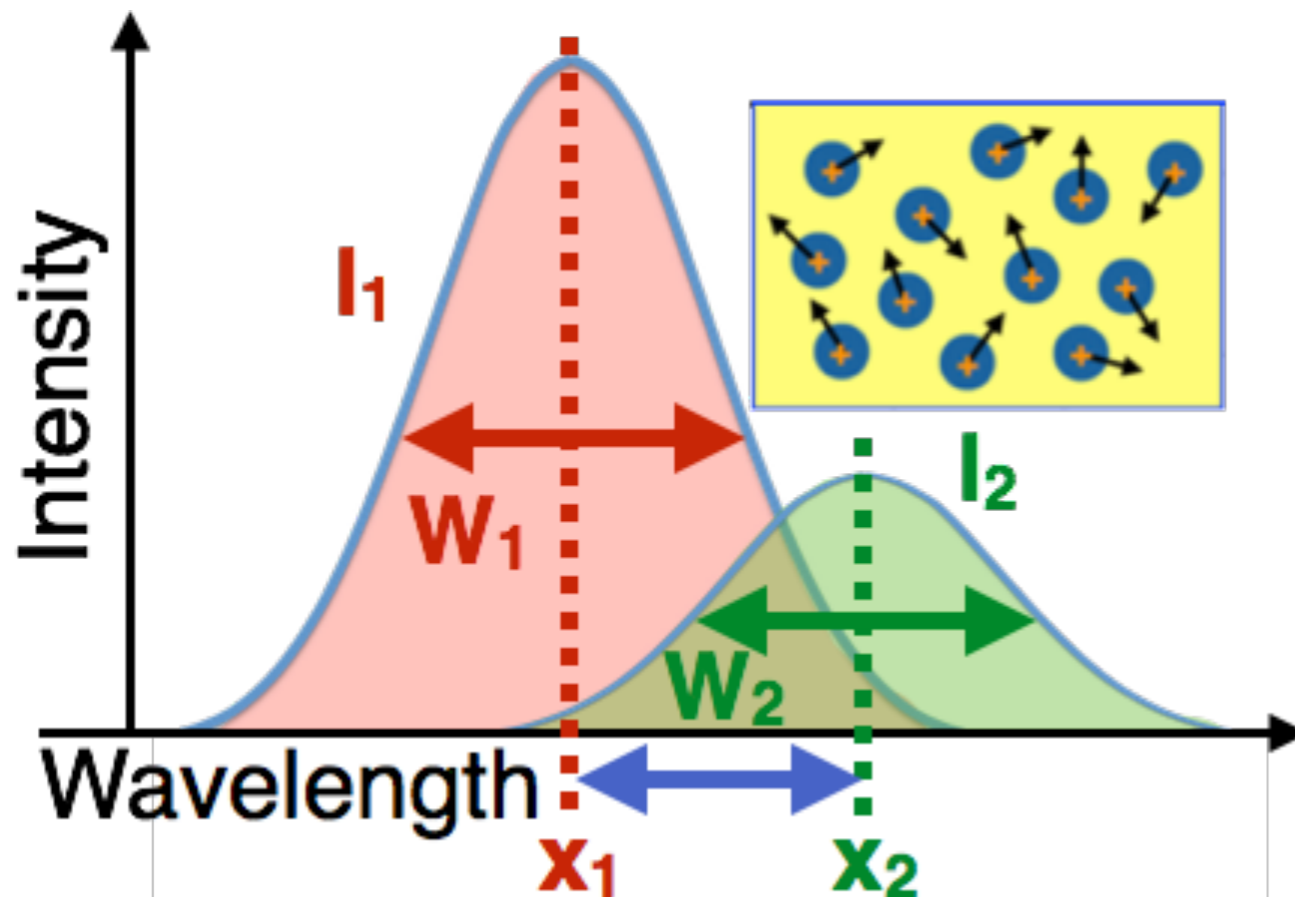


- **Turbulence** is an important mechanism for the transfer of magnetic energy e.g. Larosa & Moore 1993, Petrosian 2012, Vlahos et al. 2016, Kontar et al. 2017.
- **Turbulence** may dissipate energy over multiple fragmented regions during the flare e.g. Vlahos et al. 2016, Gordovsky et al. 2016.
- **Energy is transferred** from large scales ( $\sim$ size of a solar flare loop  $10^9$  cm) to small scales (particle level).

*How do we diagnose the presence and properties of turbulence in solar flares?*

## Spectral line broadening is the main tool

- The properties of the plasma are usually found from Gaussian fitting. The second moment is related to the width of the line (which can be converted to a velocity).
- The ‘expected’ width of a spectral line is related to temperature.
- An excess width may be due to the presence of turbulence.



- The **non-thermal velocity** is attributed to **plasma turbulence**.

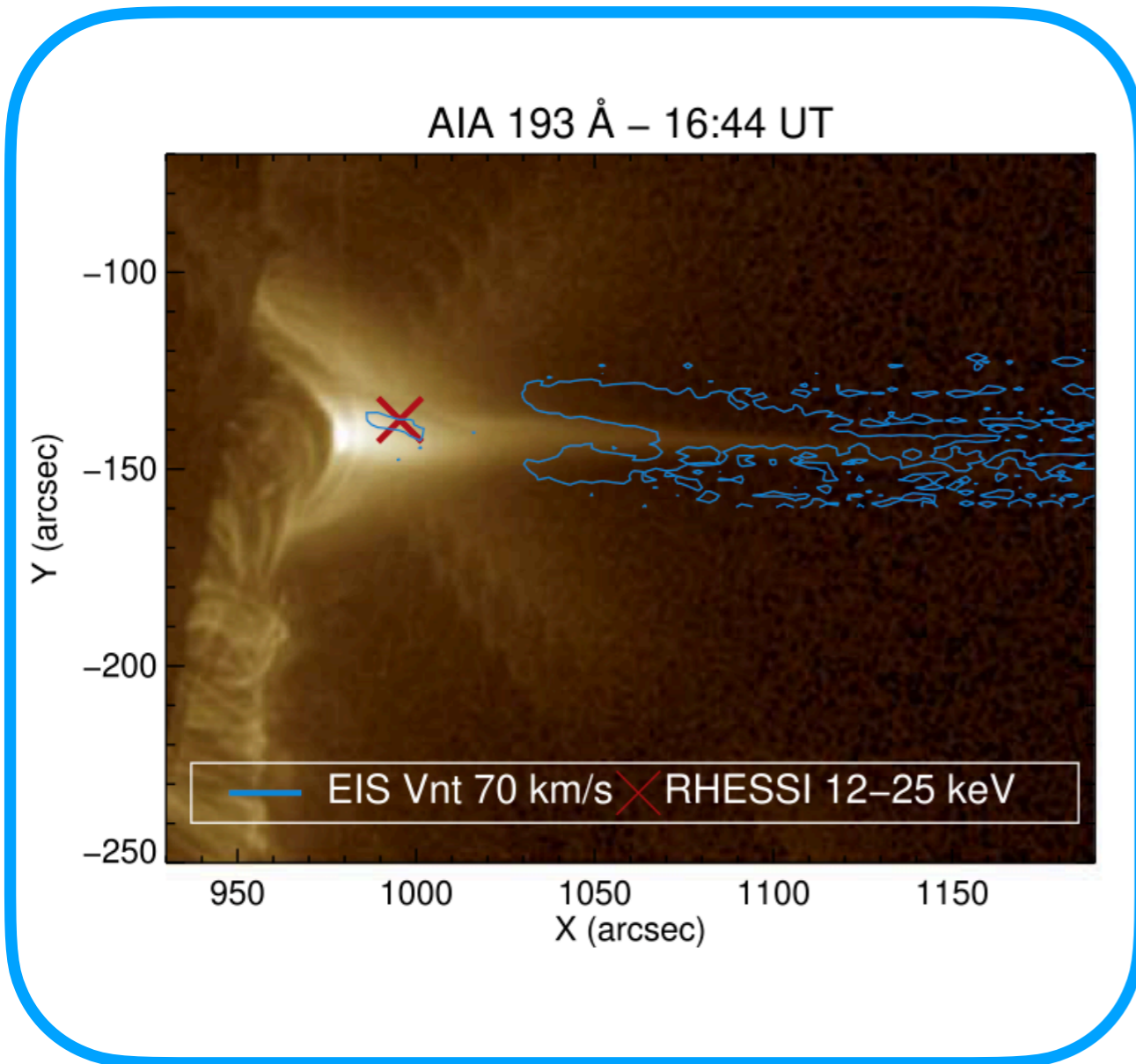
$$v = \sqrt{v_{\text{th}}^2 + v_{\text{inst}}^2 + v_{\text{nth}}^2}$$

*thermal*

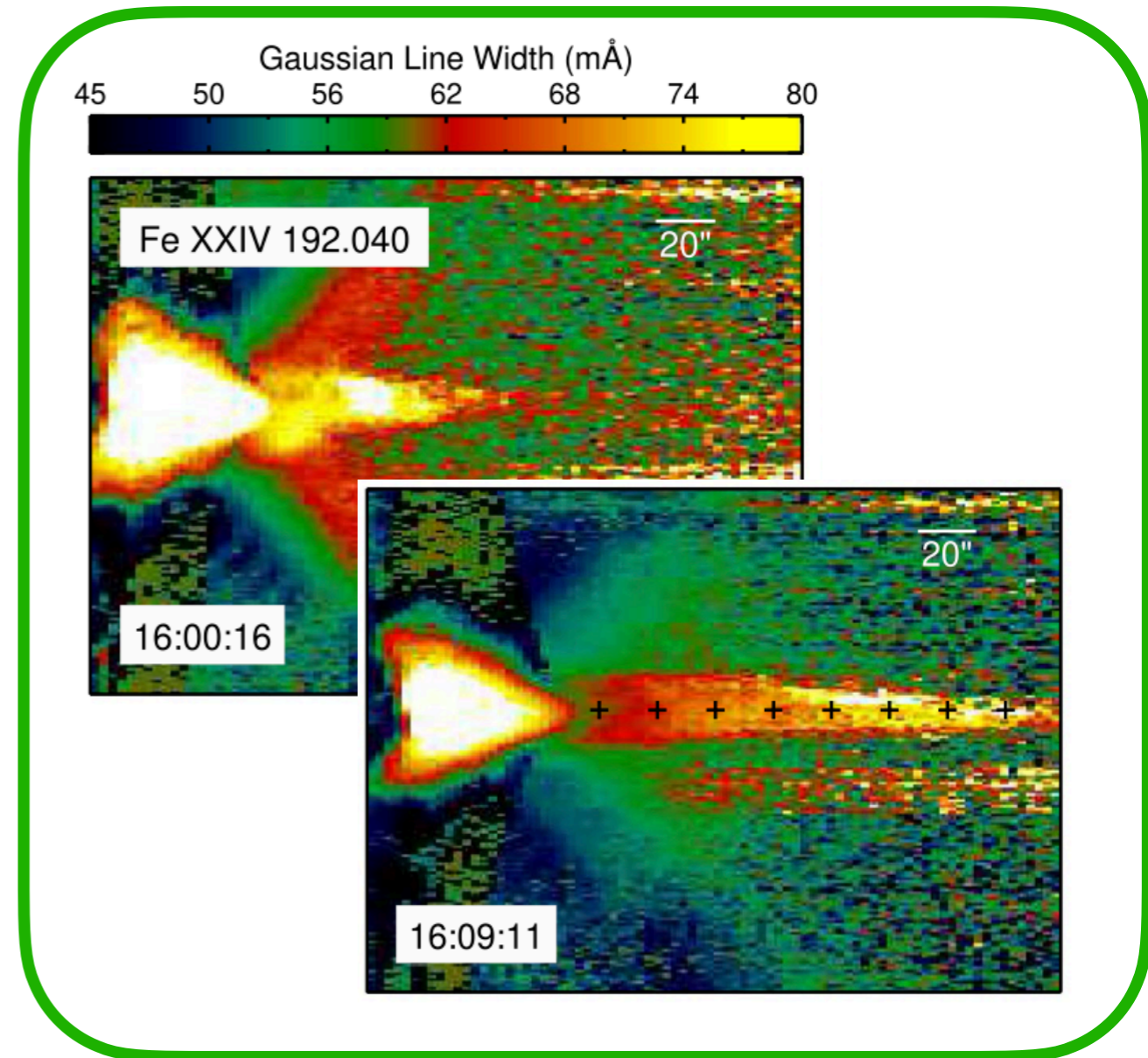
*instrument*

*nonthermal*

- **Turbulence** is usually inferred via the presence of non-thermal line broadening.



French, Matthews et al. 2020

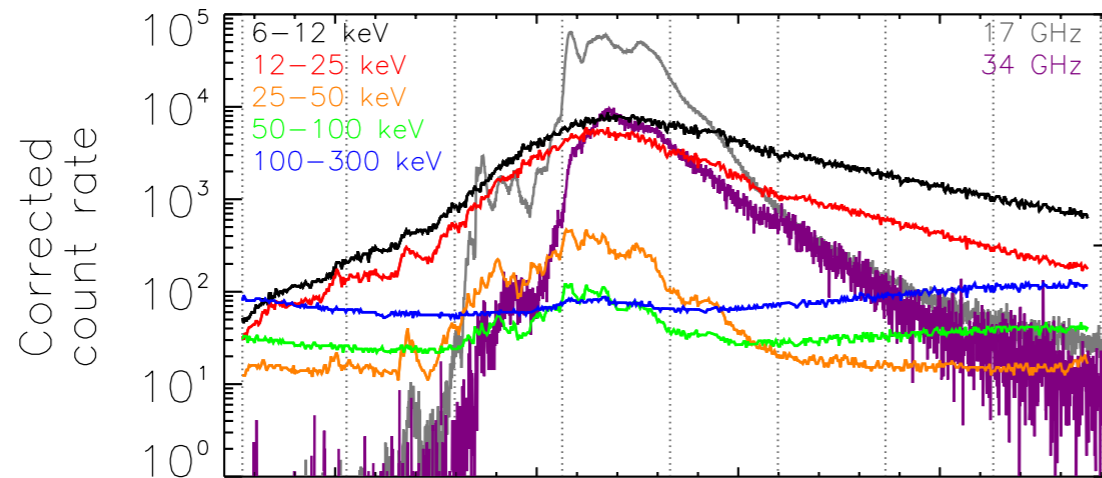


Warren, Brooks et al. 2018

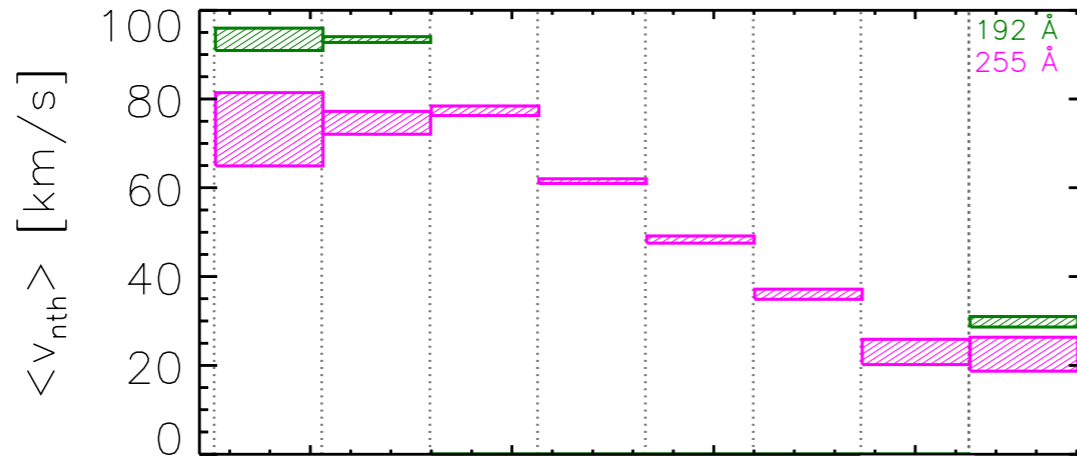
# Solar Flare Turbulence

**Turbulence plays a vital role in the transfer of energy from magnetic fields.**

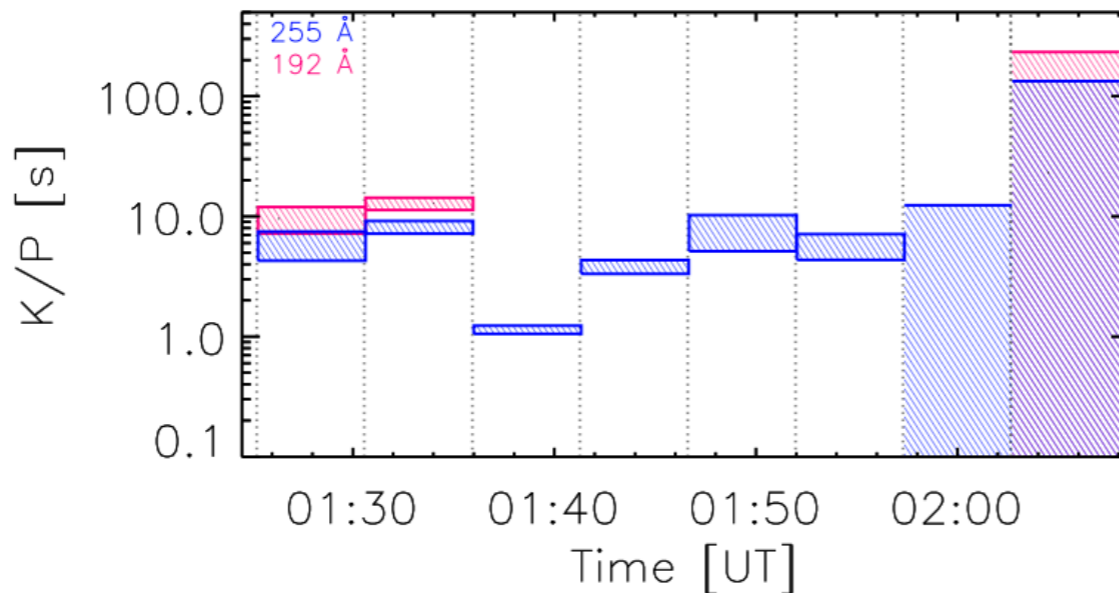
Flare Light curves



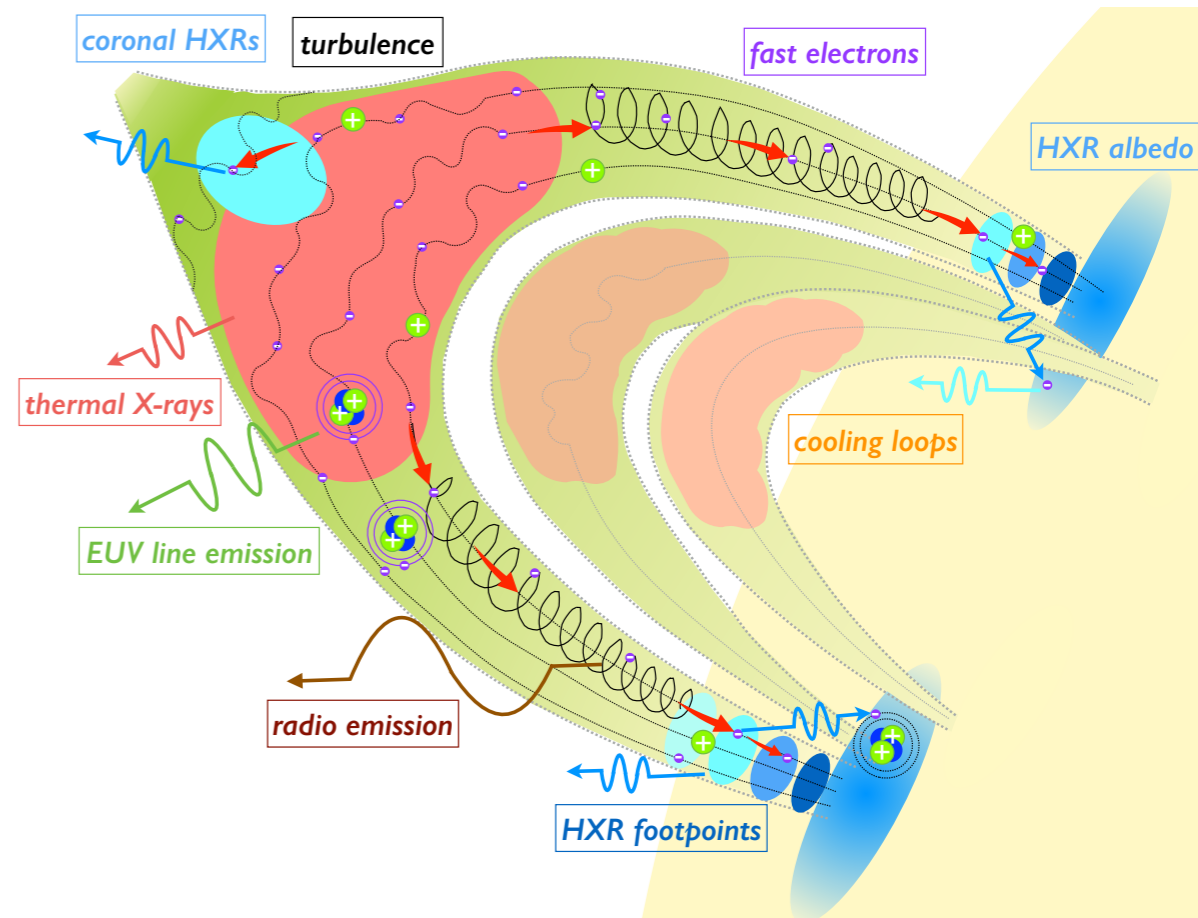
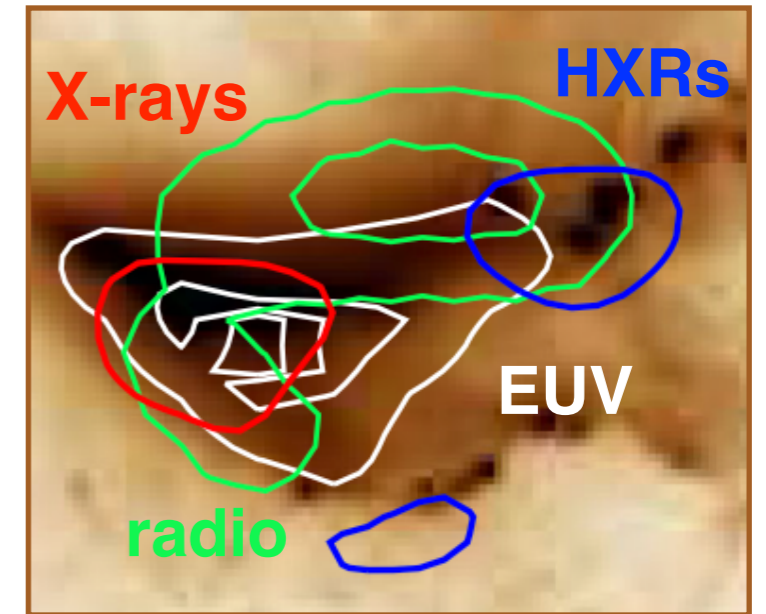
Non-thermal velocity (turbulence)



Turbulent dissipation time



Kontar et al. 2017

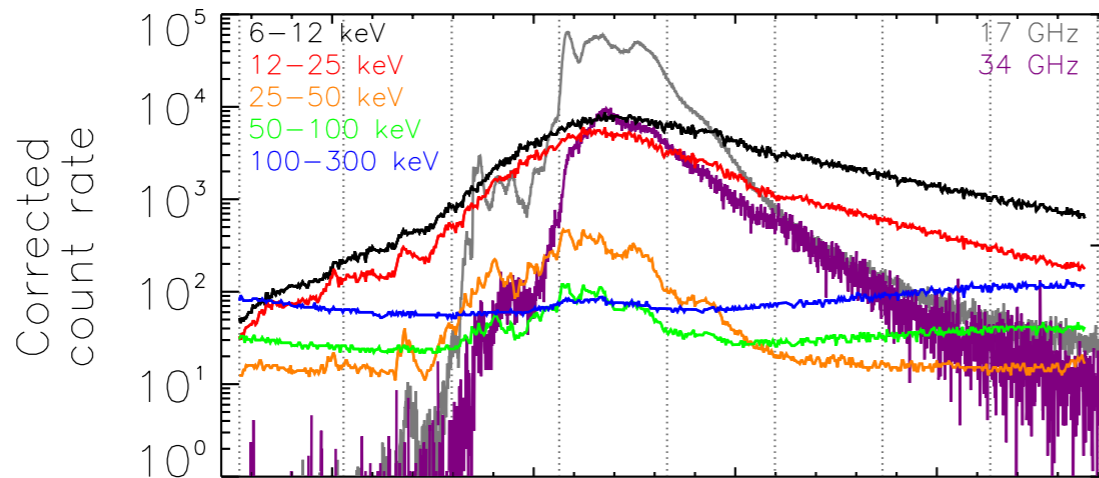




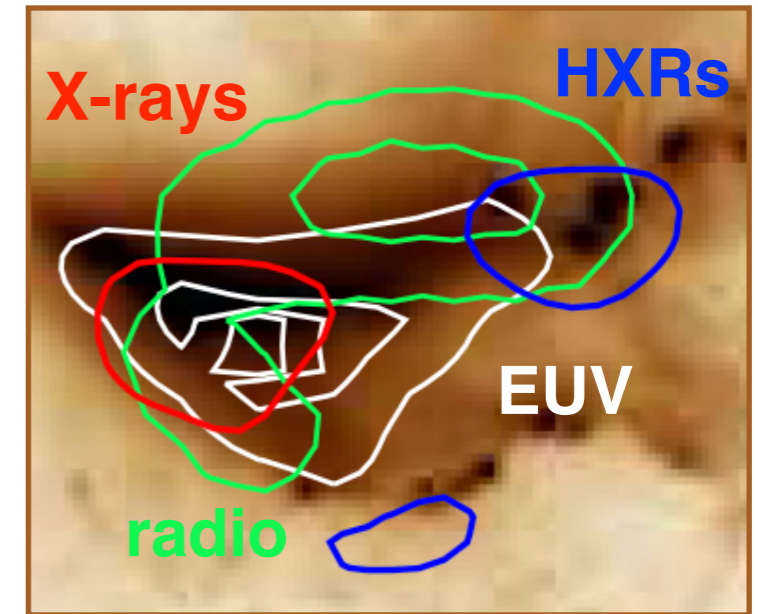
# Solar Flare Turbulence

**Turbulence plays a vital role in the transfer of energy from magnetic fields.**

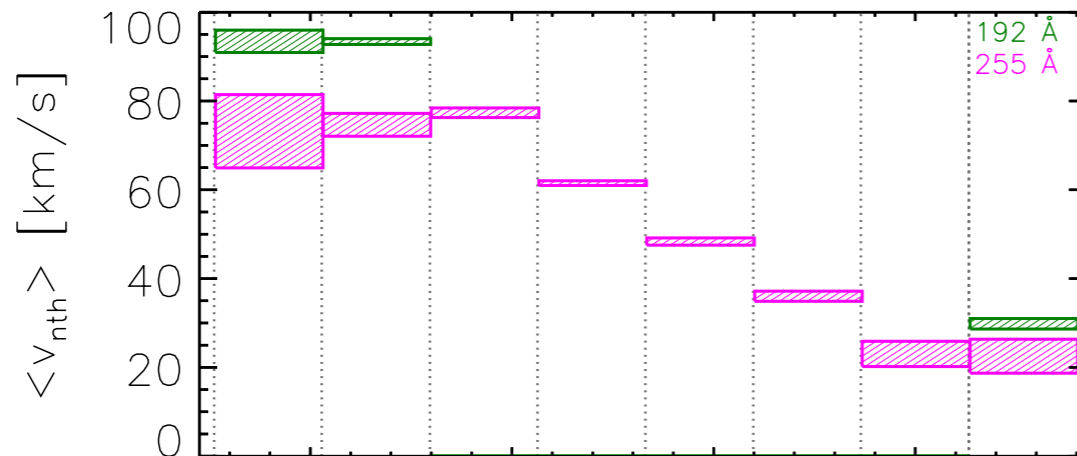
Flare  
Light  
curves



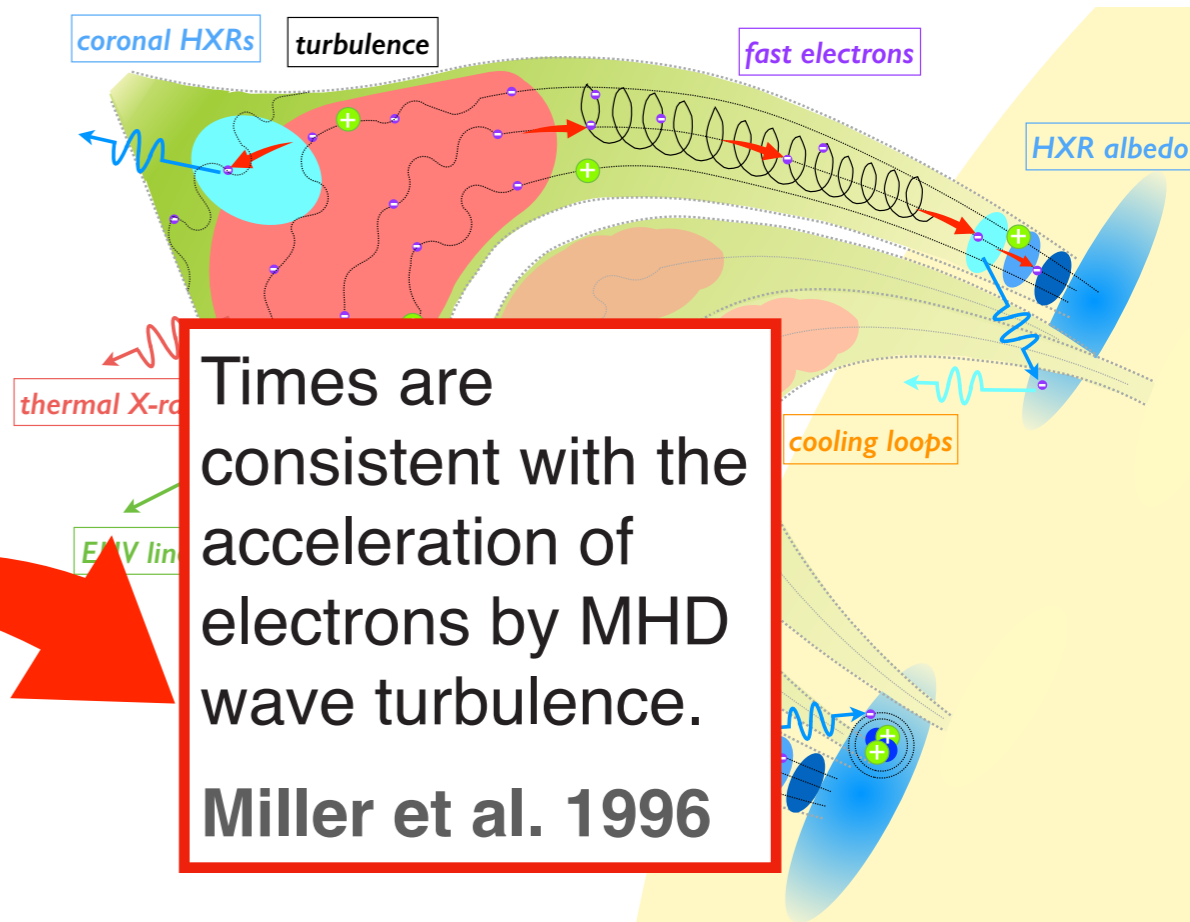
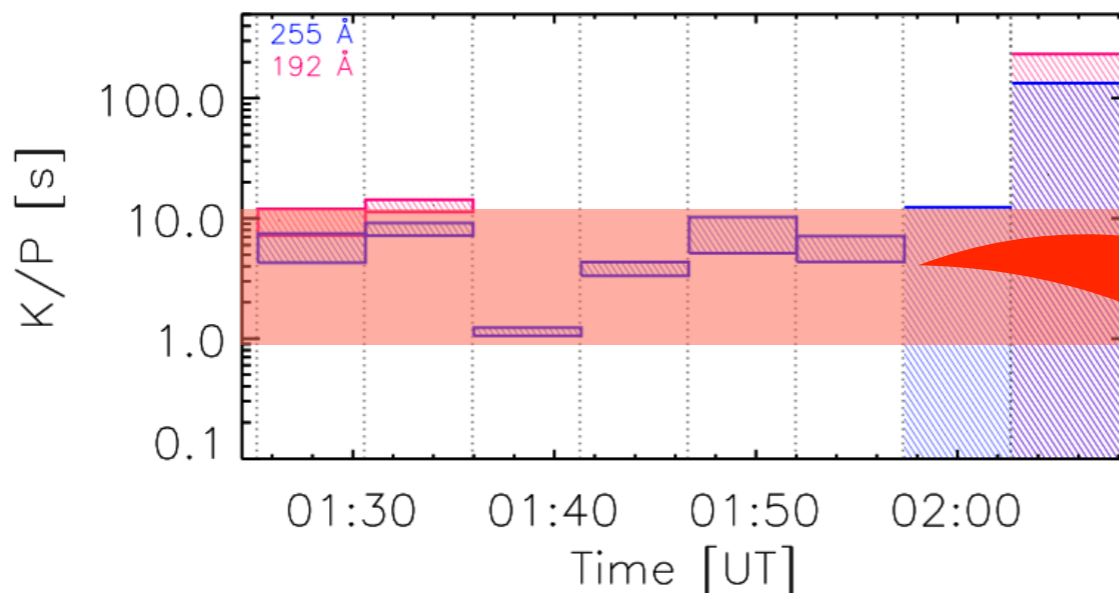
Kontar et al. 2017



Non-thermal  
velocity  
(turbulence)



Turbulent  
dissipation  
time



Times are consistent with the acceleration of electrons by MHD wave turbulence.  
Miller et al. 1996

# Solar Flare Turbulence

- *Stores et al. (2021)* provides a detailed study of flare turbulence in space

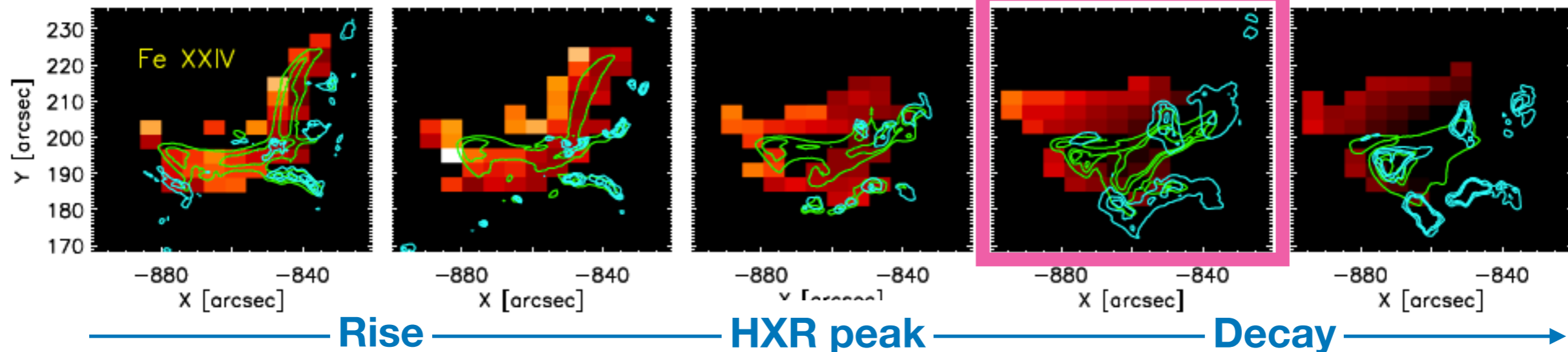
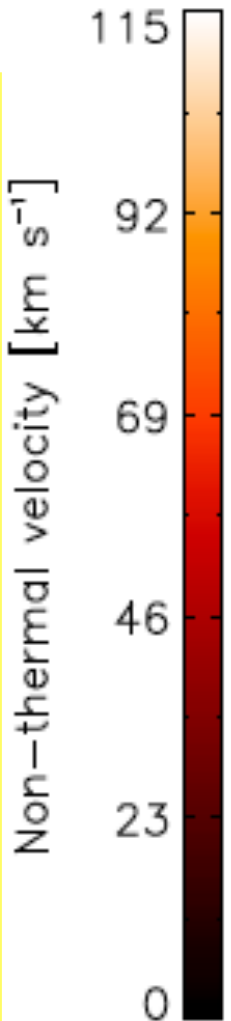
**During/After HXR peak**

Cusp/looptop

AIA 94 Å

Decreasing non-thermal line broadening

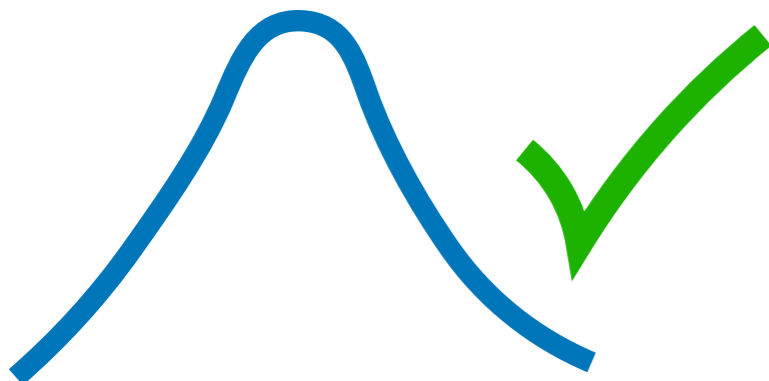
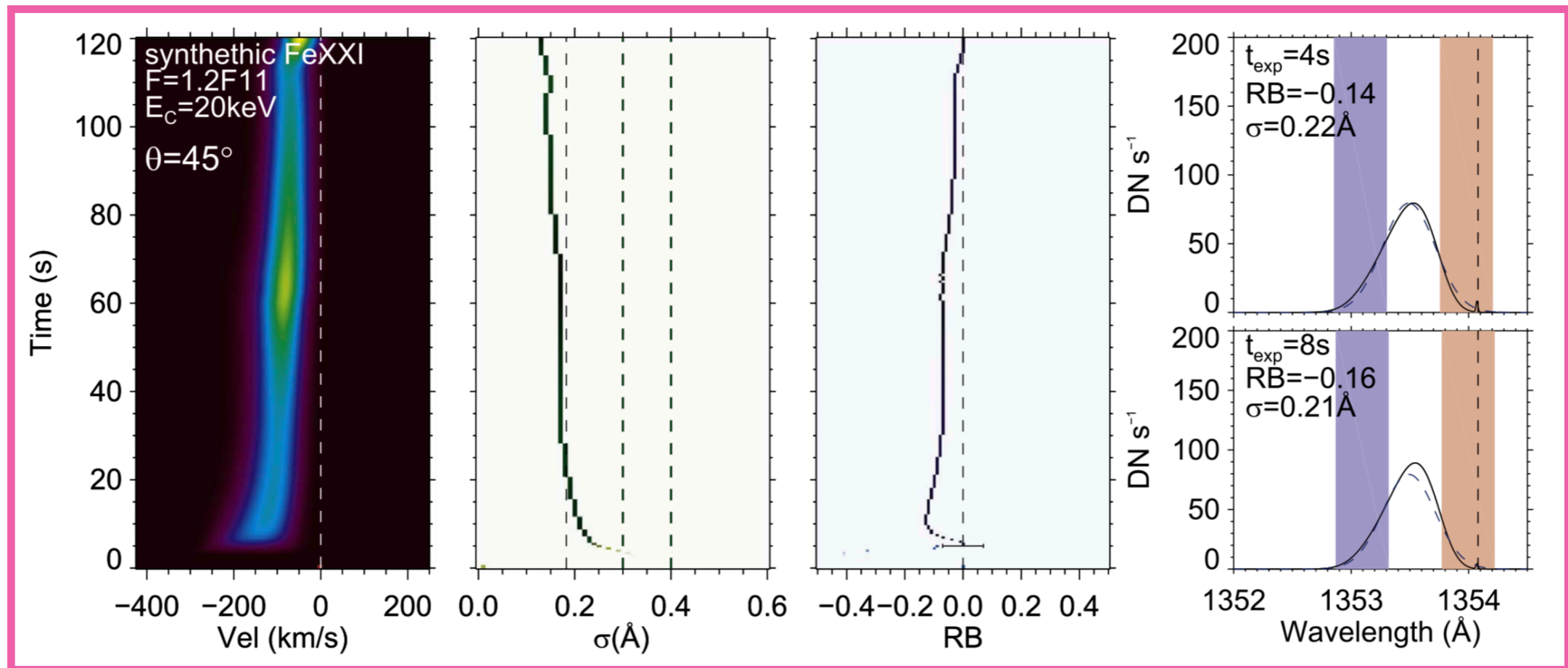
EIS Fe XXIV



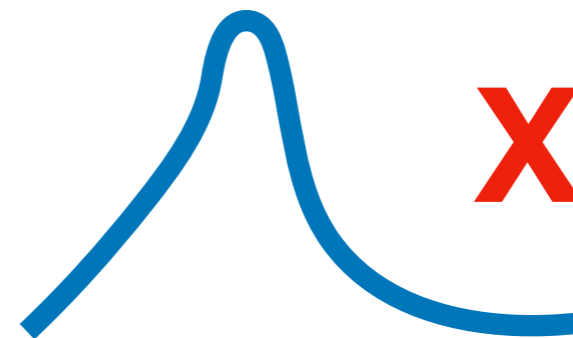
- *Stores et al. (2023)* studies electron acceleration in **extended turbulent regions**.

# Solar Flare Turbulence

- Turbulence OR superposition of unresolved plasma flows along the line of sight?
- **Polito et al. 2019** suggest that it is difficult to reconcile symmetrical broadened lines with flows (flows are more likely to produce asymmetrical broadened lines).



**More likely**  
**random plasma**  
**motions -**  
**Turbulence?**

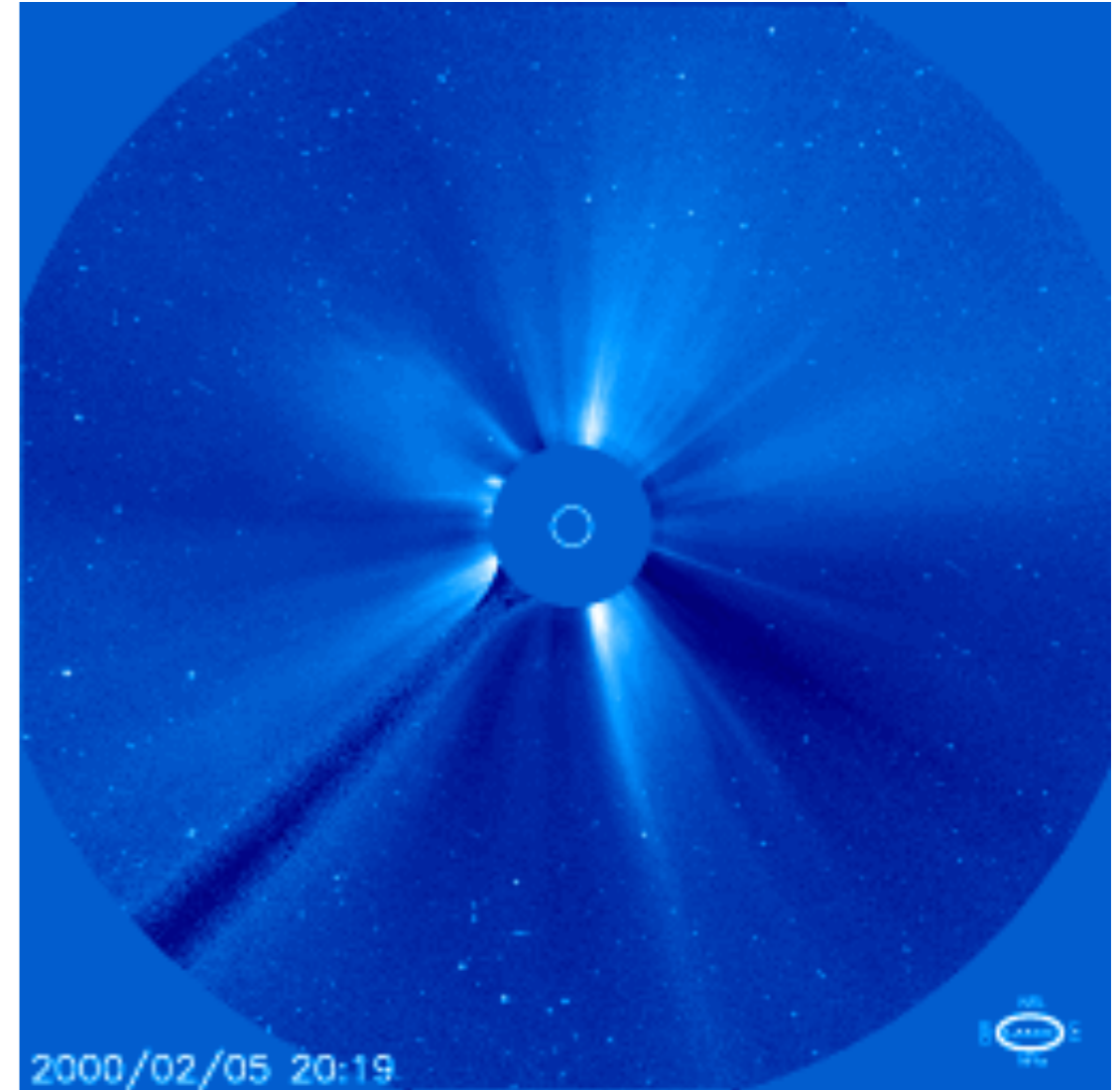


**More likely**  
**superposition of**  
**unresolved,**  
**independent flows?**

# Solar Wind Brief Recap

*See the slides from Tuesday's lecture "CME's, the Solar Wind and the Heliosphere".*

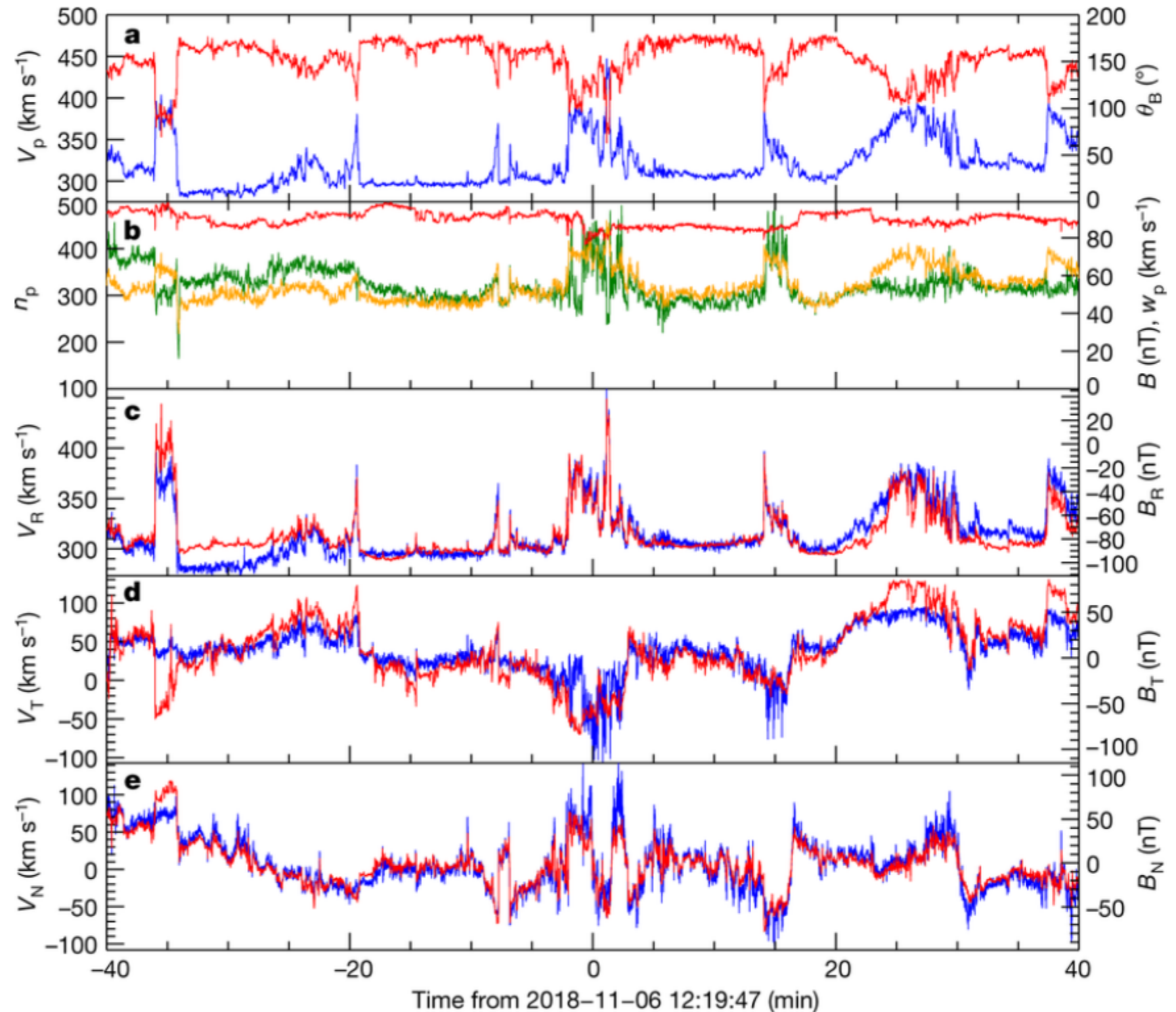
- The whole heliosphere is permeated by the solar wind, a plasma flow of solar origin that expands into the heliosphere.
- The heated corona expands into space, accelerates and forms the solar wind.
- **Solar wind properties (e.g., speed, temperature, density) are highly variable.**
- **The solar wind carries magnetic field, waves and turbulence into the heliosphere.**
- Two states: an irregular slow wind with typical speeds of 400 km/s and a smooth fast wind with a speed of  $\sim 750$  km/s.



**Turbulence in the solar heliosphere plays a relevant role in several aspects of plasma behaviour in space, such as solar wind generation, high-energy particles acceleration, plasma heating, and cosmic rays propagation.**

# Solar wind turbulence

- The solar wind is a highly turbulent plasma, and hence, an excellent laboratory for studying turbulence!
- In the solar wind, **turbulence can be measured directly via in-situ measurements at different heliospheric locations** e.g., fluctuating B-fields, E-fields.



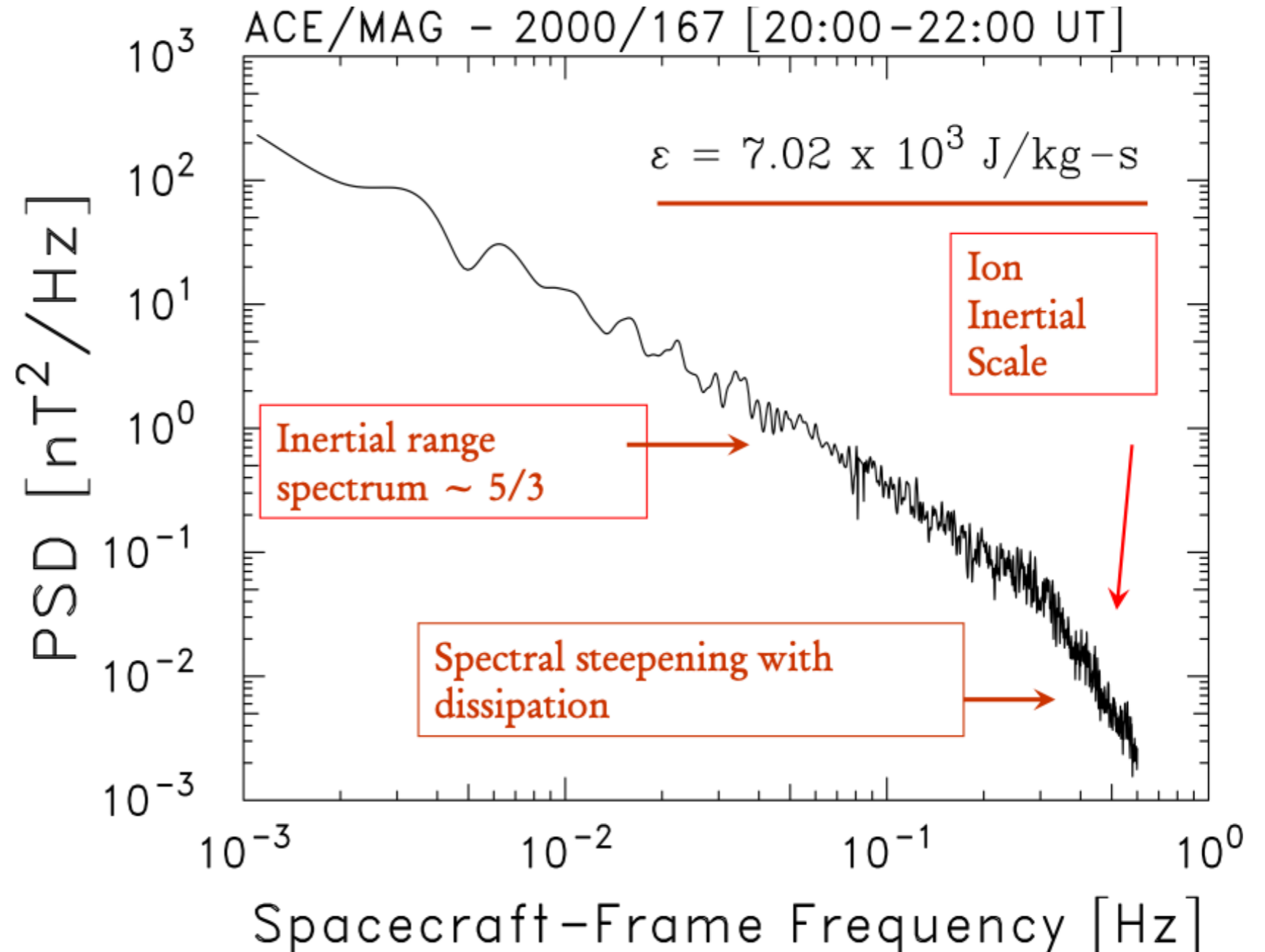
*PSP observations, Kasper et al. (2019), Nature, 576, 228.*

- Real time solar wind: <https://www.swpc.noaa.gov/products/real-time-solar-wind>

# Solar wind turbulent cascade

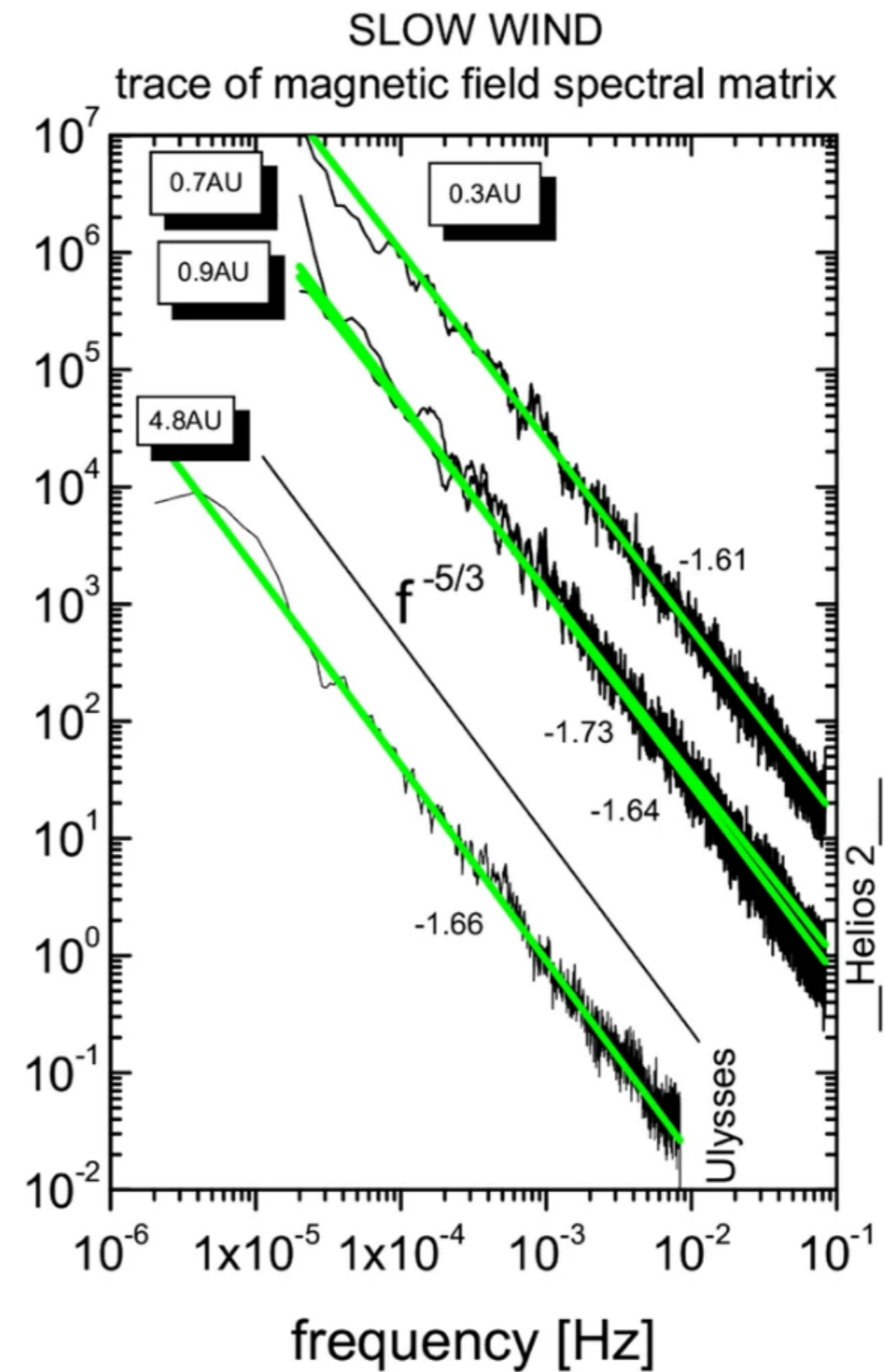
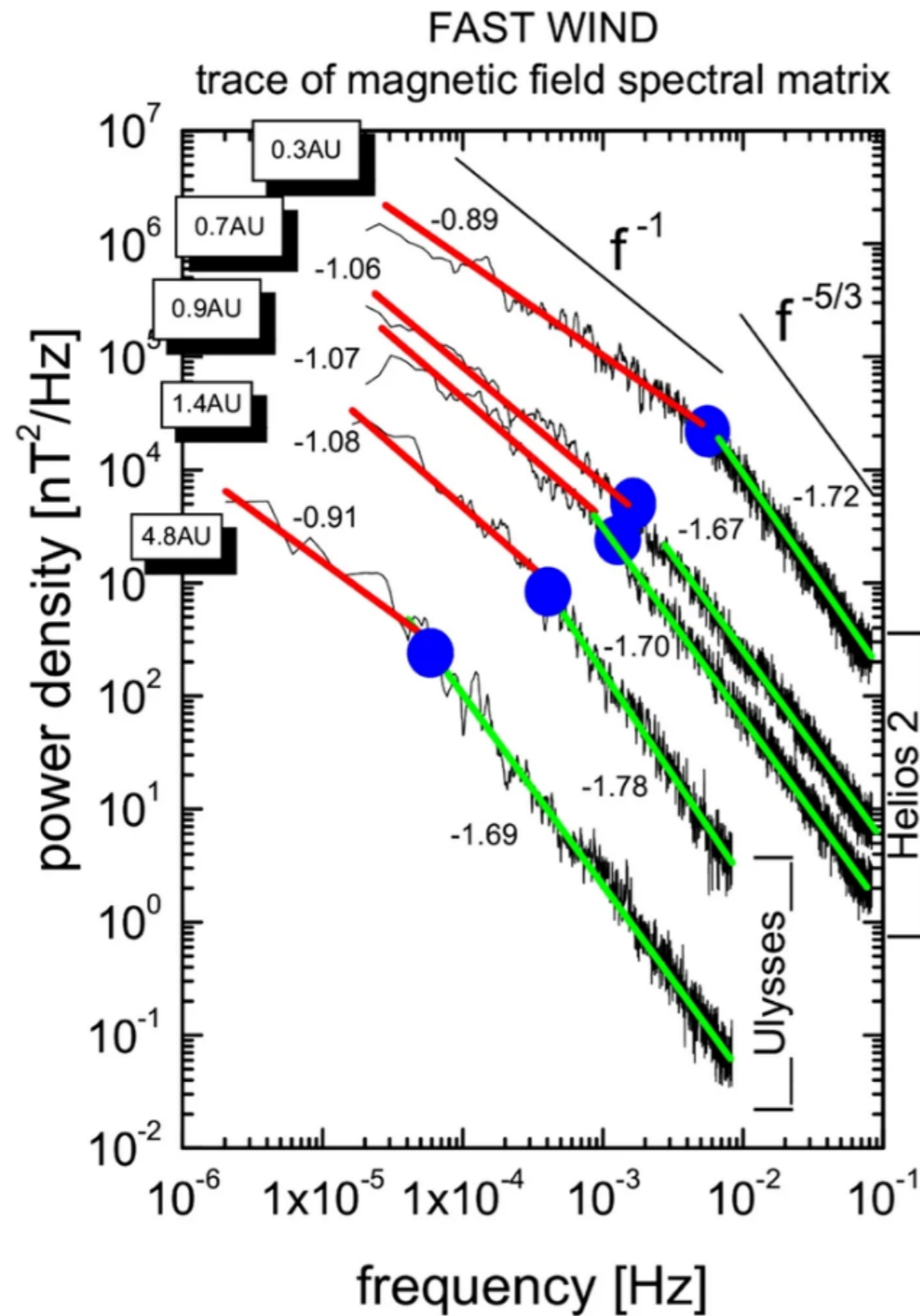
- **Large-scale disturbances (shocks, ejecta, heliospheric current sheets, stream interactions) provide energy to drive the turbulent cascade.**

- The spectrum (e.g., turbulent magnetic field) displays fluctuations over several decades in scale.
- The spectral break indicates the beginning of the ion kinetic range where the ion scale lengths are of comparable sizes to the magnetic fluctuations.

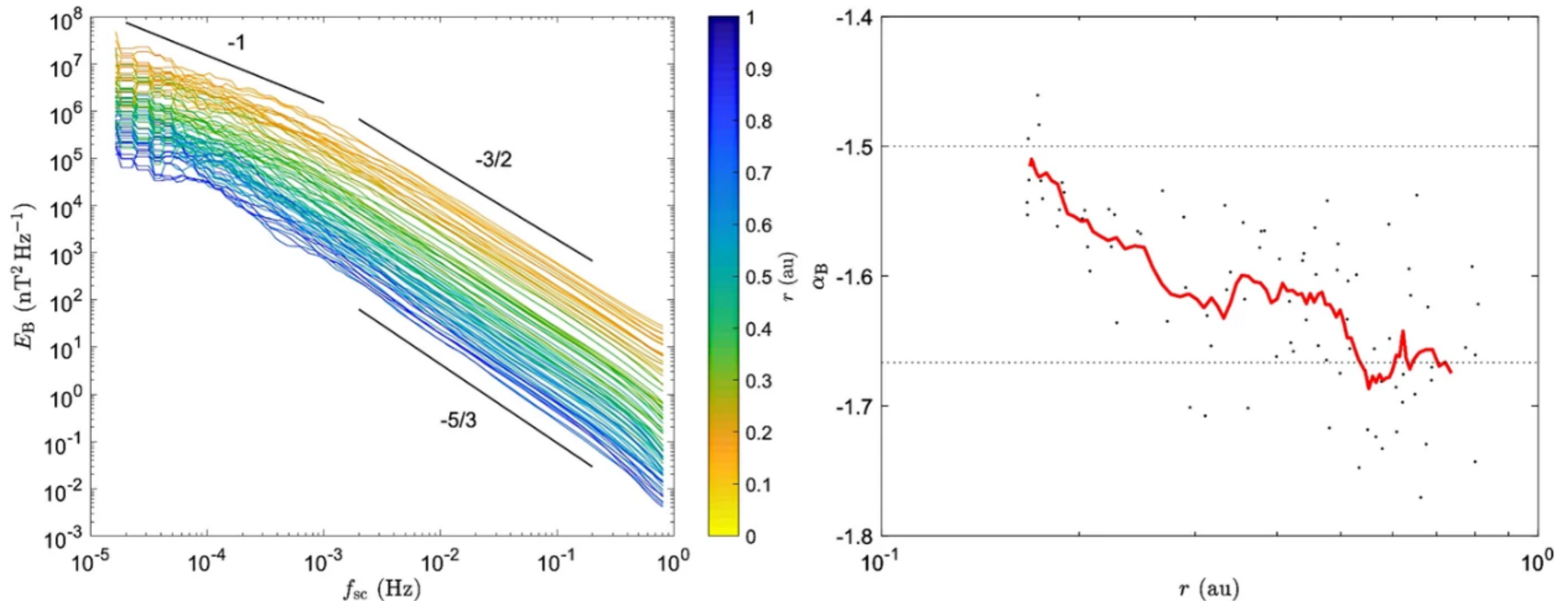


- How energy is dissipated exactly (e.g., collisionless plasma) is an open question!

- **Example:** distance differences and fast and slow wind differences.



- **Example:** PSP measurements at different radial distances in the heliosphere.



*Chen et al. (2020)*

- Net power in the magnetic fluctuations is seen to rise sunwards.
- Most of the spectra show a  $1/f$  range at the largest scales.
- Inertial range shows a transition ( $-3/2$  to  $-5/3$ ) as the radial distance increases.



- **Why do we see a Kolmogorov-type spectrum in the solar wind??????**
- Although we see IK  $-3/2$  spectra, many solar<sub>⊥</sub>wind observations (and numerical simulations) show the Kolmogorov  $-5/3$  spectrum.
- The direction of the motion of the solar wind is not constant and the orientation of the wave vectors, i.e.,  $k_{||}$  (parallel to B) and  $k_{\perp}$  (perpendicular to B) with respect to the solar wind velocity should be taken into account.
- **Weak turbulence:** many collisions are required to transfer energy to smaller scales leading to a slower cascade rate (this assumption is used in the IK spectrum).
- **Strong turbulence: energy is transferred to smaller scales in a single collision.**

$$\tau_c \approx \frac{1}{k_{||} v_A}$$

**Goldreich & Sridhar (1995) invoke strong turbulence where fluctuations dominate the mean magnetic field (critical balance theory).**

- Why do we see a Kolmogorov-type spectrum in the solar wind??????
- In Goldreich & Sridhar (1995), the assumption of **strong turbulence is assumed**.
- This leads to a nonlinear transfer time  $\tau_{NL}$  approximately equal to the collision time  $\tau_c$ .

$$\tau_{nl} \approx \tau_c \rightarrow \frac{1}{k_{||} v_A} \approx \frac{1}{k_{\perp} v_{\perp}}$$

- This is known as **Critical Balance**.

- Perpendicular transfer rate:  $\epsilon = \frac{\text{energy}}{\text{time}} \approx k_{\perp} v_{\perp}^3 \quad v_{\perp} \approx \epsilon^{1/3} k_{\perp}^{-1/3}$

**Kolmogorov spectrum**

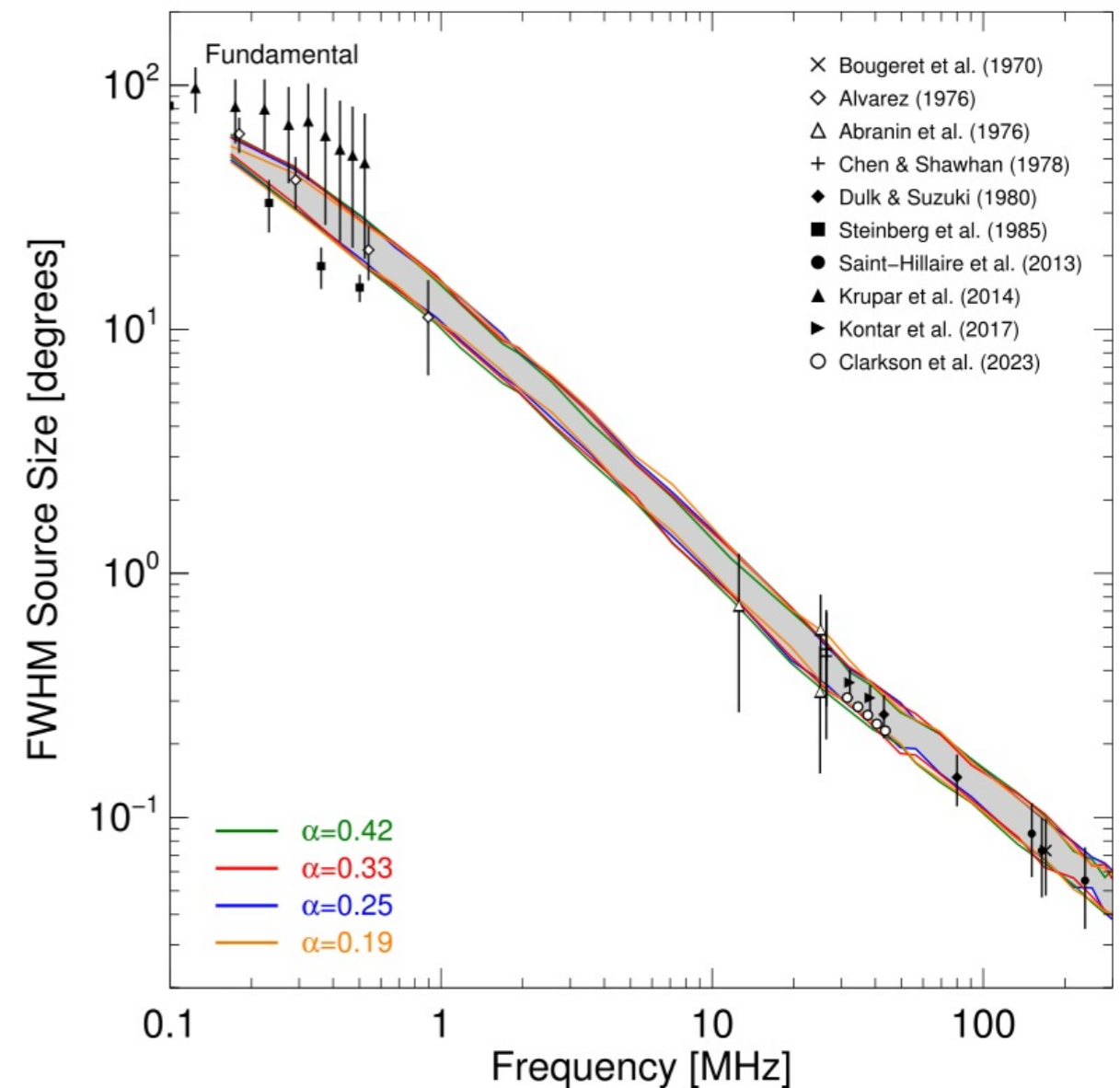
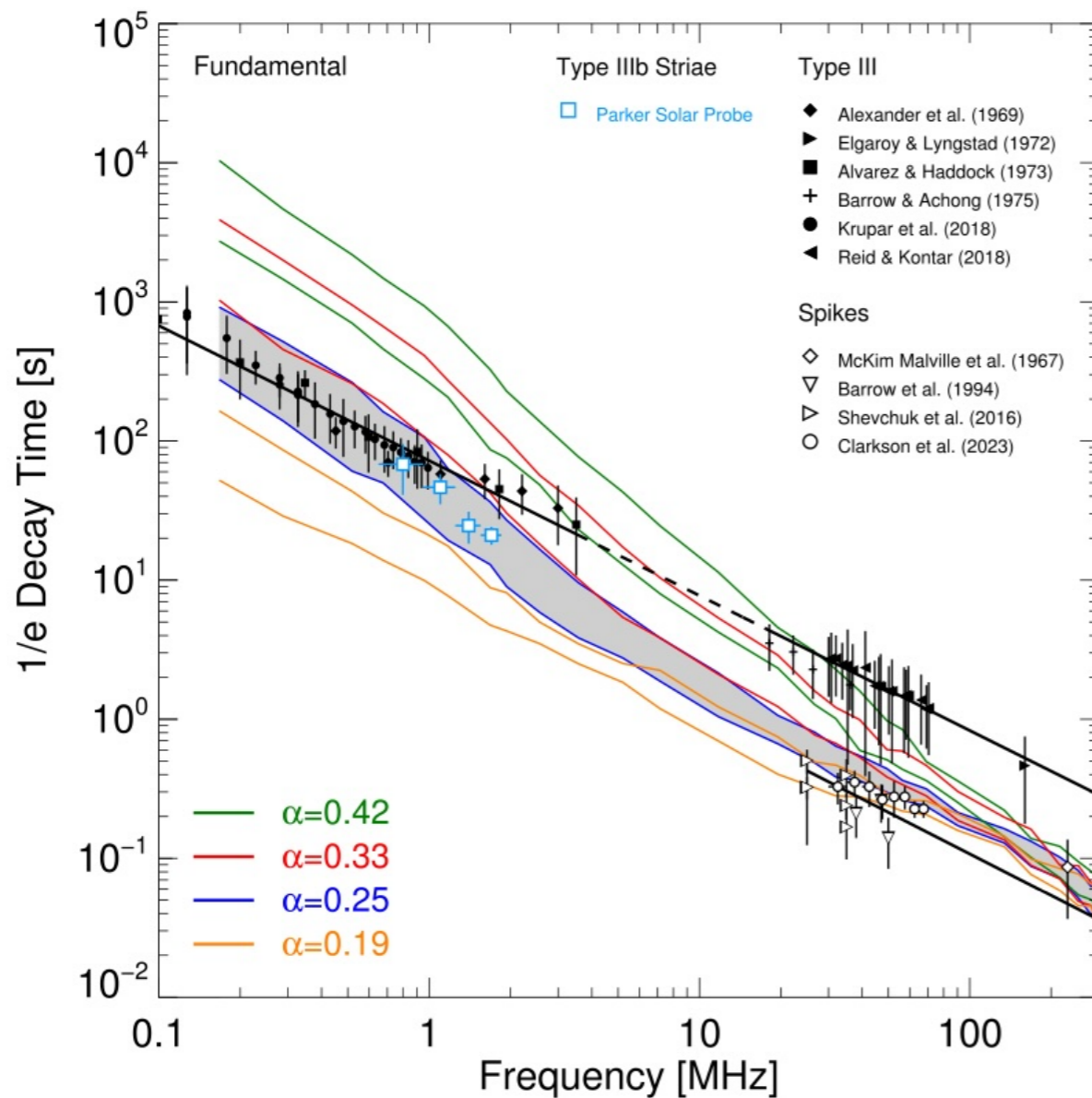
$$E(k_{\perp}) \approx k_{\perp}^{-5/3}$$

- Parallel transfer rate:  $k_{||} v_A \approx k_{\perp} v_{\perp} \approx k_{\perp} \left( \epsilon^{1/3} k_{\perp}^{-1/3} \right) \approx \epsilon^{1/3} k_{\perp}^{2/3} \quad k_{||} \propto k_{\perp}^{2/3}$

**Parallel spectrum**

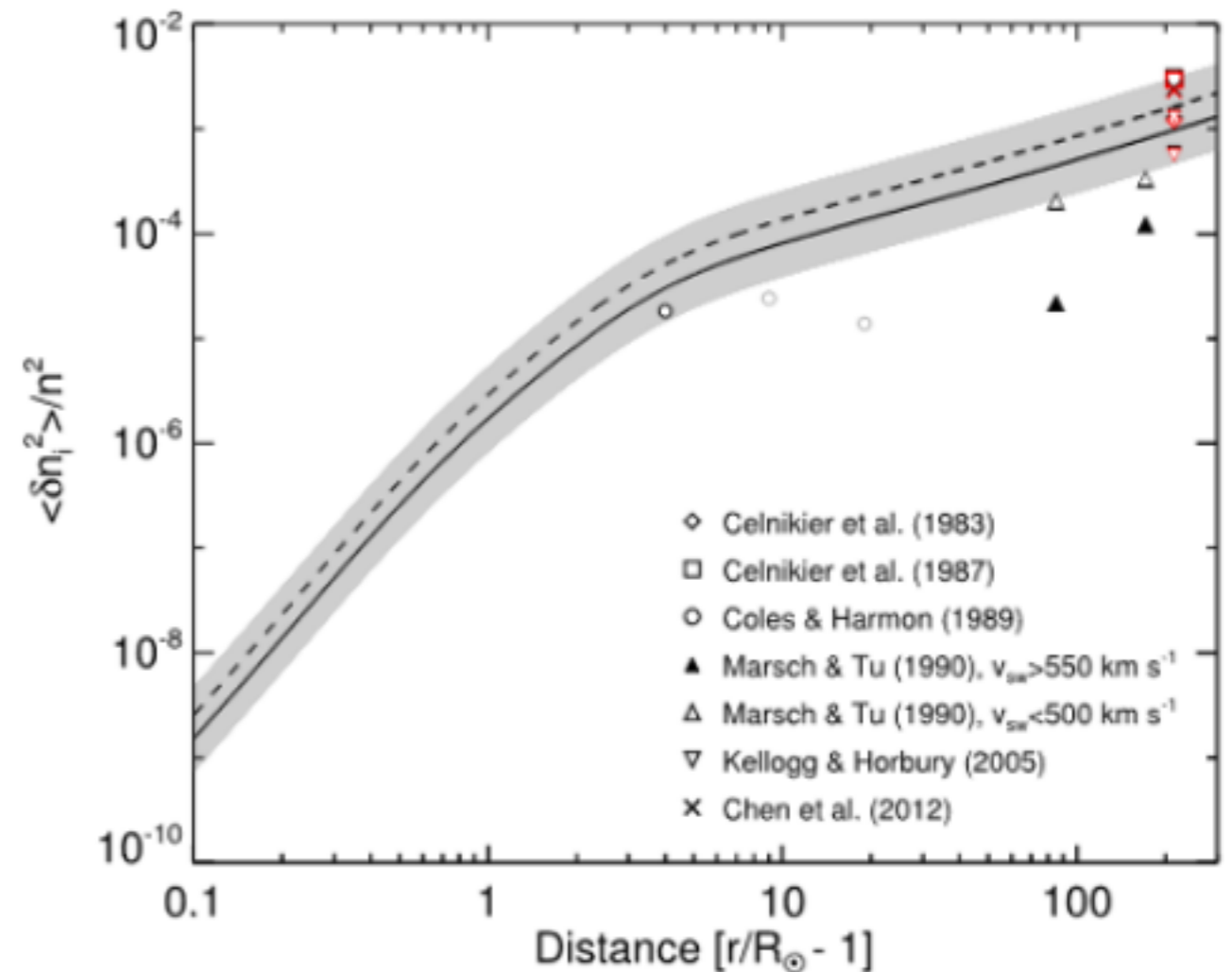
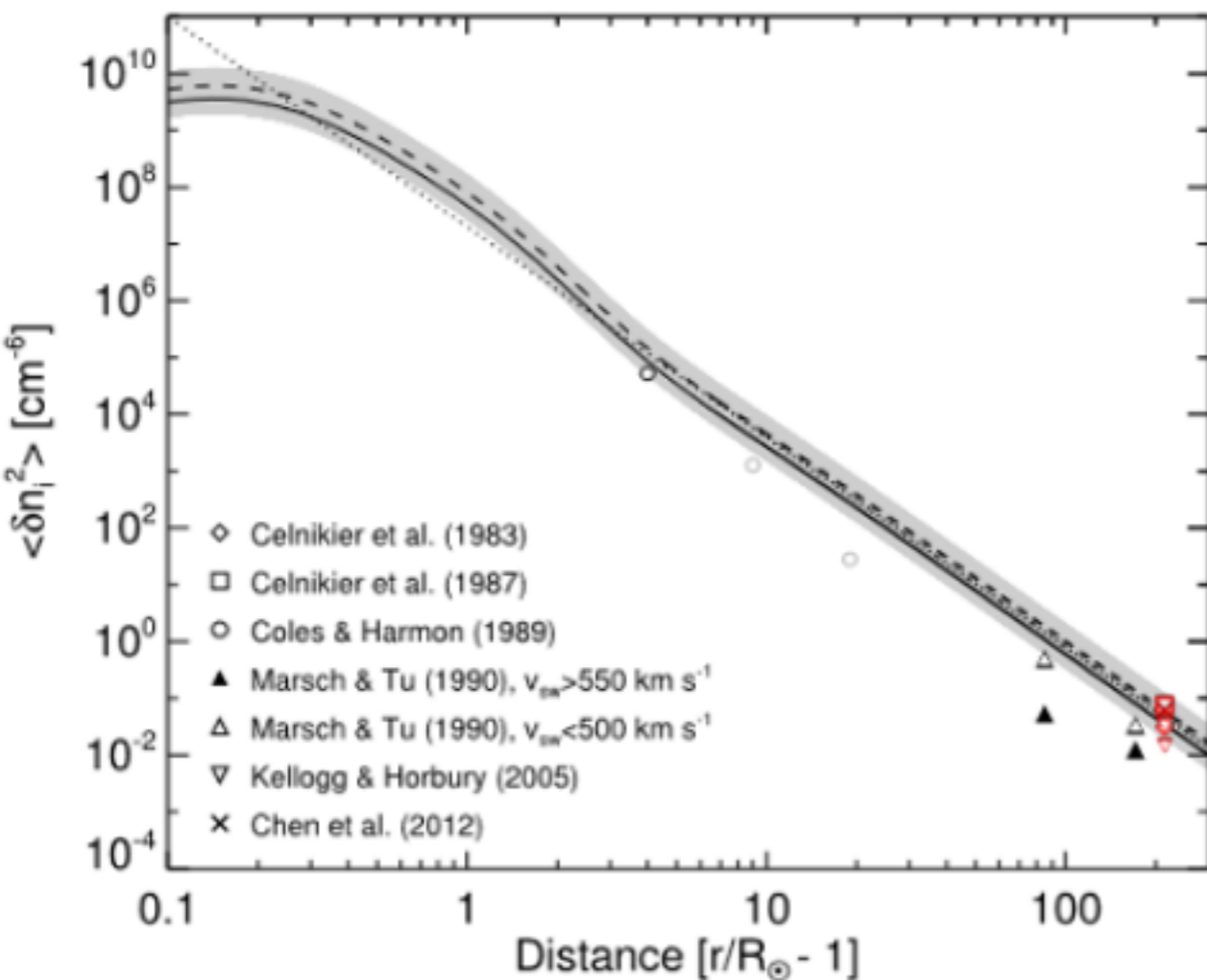
$$E(k_{||}) \approx k_{||}^{-2}$$

- Density fluctuations are not as well studied as magnetic field fluctuations.
- Novel radio observations can be used to map turbulence via density fluctuations over large distance in the heliosphere (from the Sun to 1 AU).



- Density fluctuations are not as well studied as magnetic field fluctuations.
- Matching of simulations with radio observations suggests that the turbulence is anisotropic.

$$\overline{q\epsilon^2}(r) R_\odot n^2 \simeq 6.5 \times 10^{14} (r/R_\odot - 1)^{-5.17} \quad \text{where} \quad \overline{q\epsilon^2} \sim 0.5-2.$$



**Turbulence plays an important role in most astrophysical systems including in solar and space physics.**

- **Open questions/problems:**

- MHD turbulence is believed to be important for coronal heating problem and acceleration of the solar wind.
- MHD turbulence plays a key role in energy transfer in solar flares, coronal mass ejections, magnetic reconnection at the Sun and in a variety of astrophysical phenomena.
- The details of development and dissipation of MHD turbulence are not fully understood!

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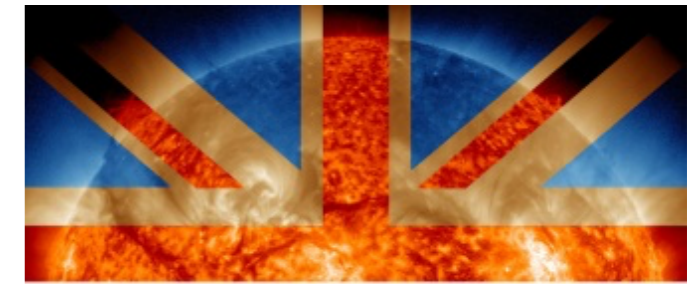
Stores et al. 2023 <https://ui.adsabs.harvard.edu/abs/2023ApJ...946...53S/abstract>

Warmuth & Mann 2016 <https://ui.adsabs.harvard.edu/abs/2016A&A...588A.116W/abstract>

Warren, Brooks et al. 2018 <https://ui.adsabs.harvard.edu/abs/2018ApJ...854..122W/abstract>

Vlahos et al. 2016 <https://ui.adsabs.harvard.edu/abs/2016ApJ...827L...3V/abstract>

- UK Solar Physics are a specialist group affiliated to the Royal Astronomical Society (RAS). The council represents the solar community in the UK and perform tasks beneficial to that community such as providing a newsletter (with workshops, jobs etc.), organising solar sessions at NAM and liaising between the solar community and other similar groups such as MIST (Magnetosphere, Ionosphere and Solar-Terrestrial).



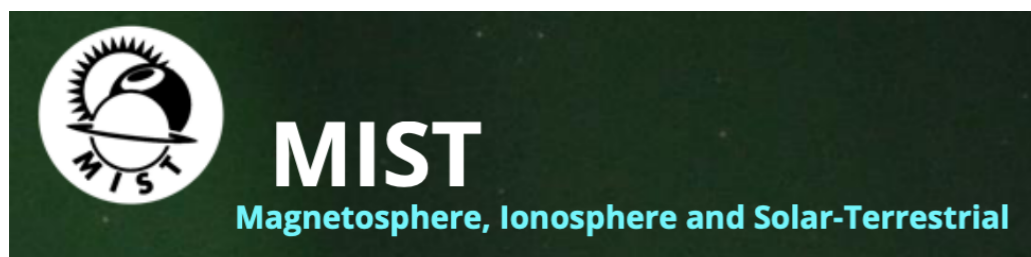
UKSP

<https://www.uksolphys.org>

- The council has eight members covering different career levels and institutes within all four parts of the UK: Natasha Jeffrey (chair), Marianna Korsos (deputy chair), Karen Meyer, Peter Wyper, Ryan Milligan, Suzana de Souza e Almeida Silva, Rahul Sharma, Matthew Lennard (PhD). A new council is elected every three years.

Please join the mailing list here: <https://www.jiscmail.ac.uk/uksp>

- MIST (Magnetosphere, Ionosphere and Solar-Terrestrial) is a similar specialist group.



<https://www.mist.ac.uk>