Introduction to Plasma Physics

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What is plasma?

“Plasma is often called “the fourth state of matter,” along with solid, liquid and gas. Just as a liquid will boil, changing into a gas when energy is added, heating a gas will form a plasma – a soup of positively charged particles (ions) and negatively charged particles (electrons).”

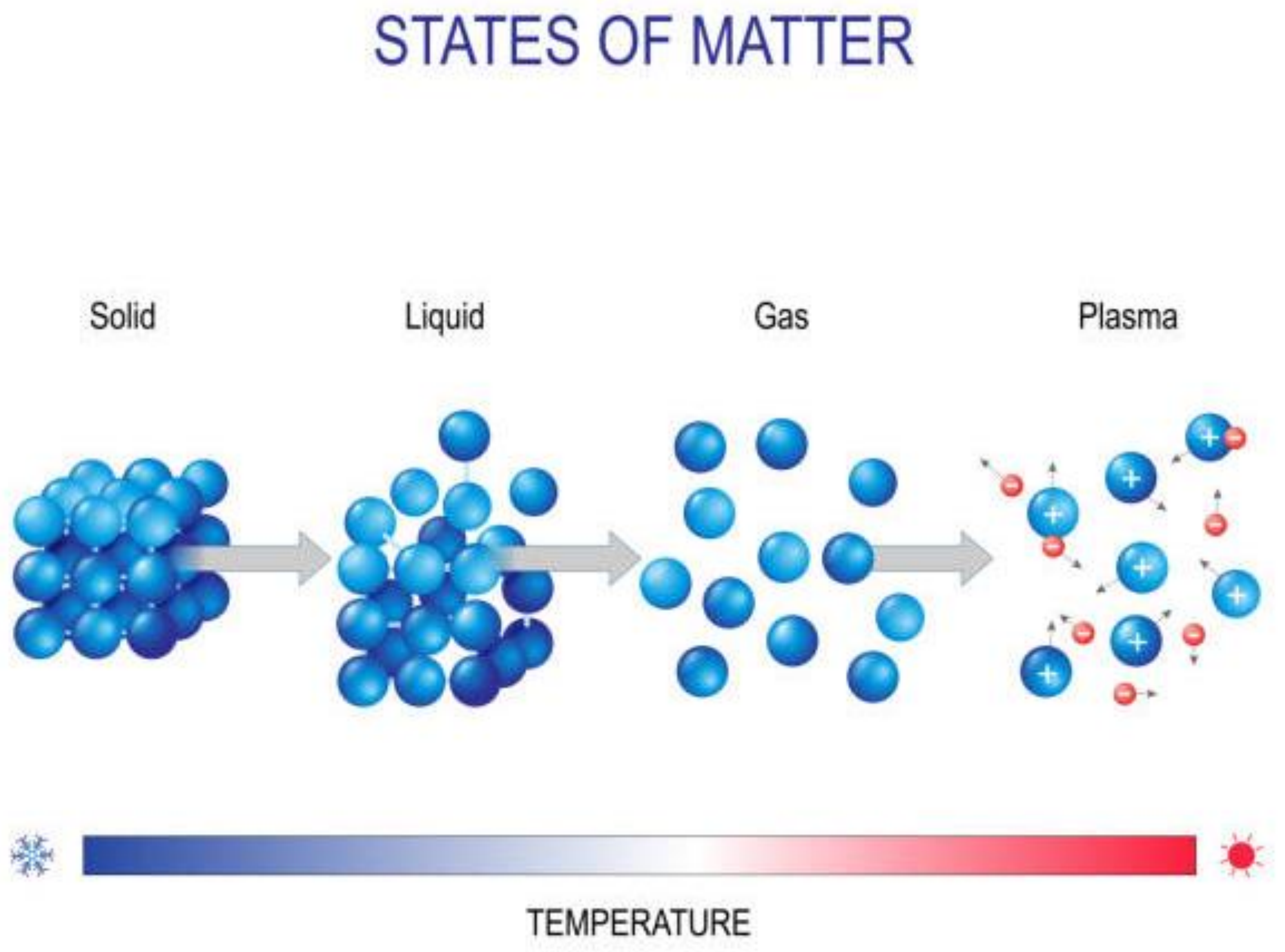


Image from https://media.istockphoto.com/

How to make a Plasma

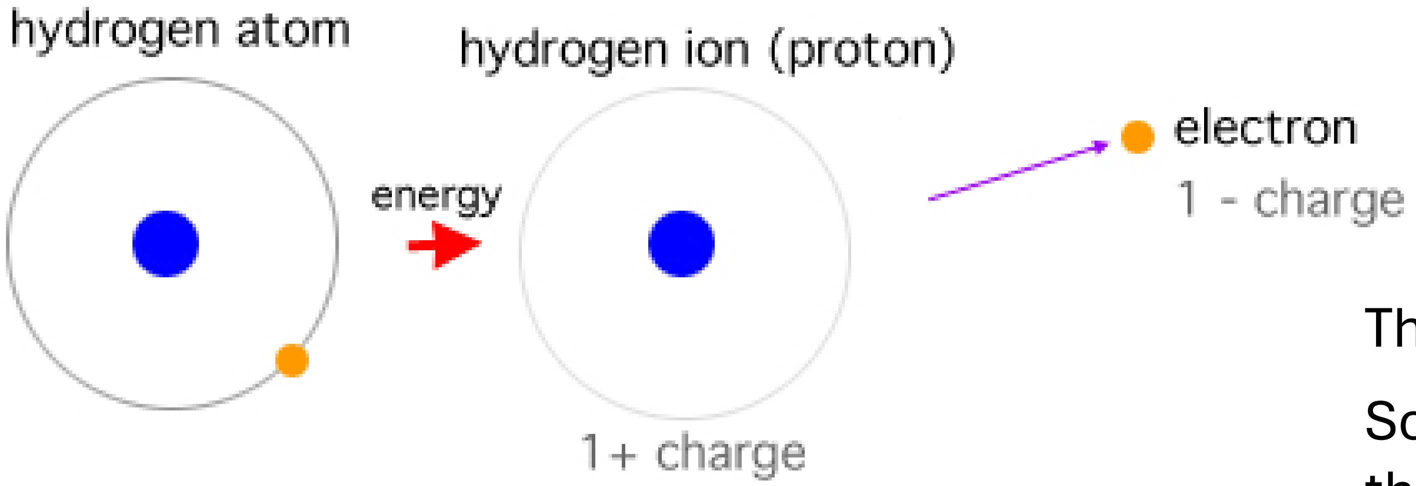


Image from http://sciexplorer.blogspot.com/

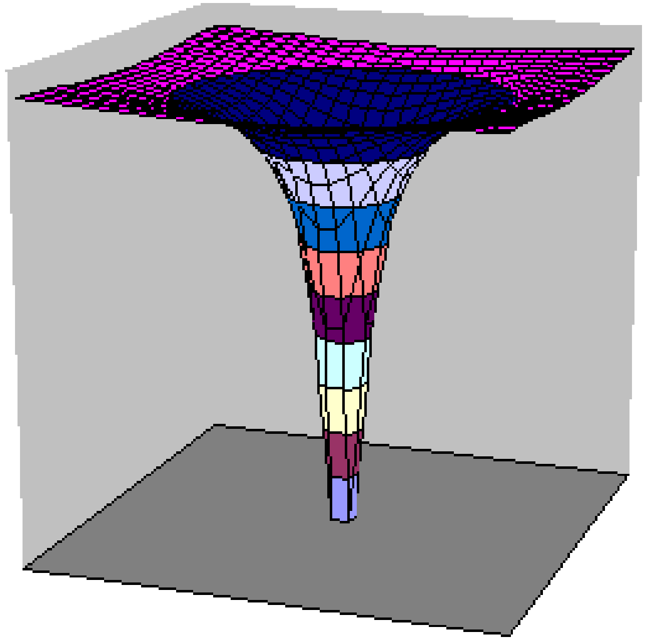


Image from http://labman.phys.utk.edu/

The potential of an atom is $U = -\frac{q_e^2}{(4\pi\varepsilon_0 r)}$.

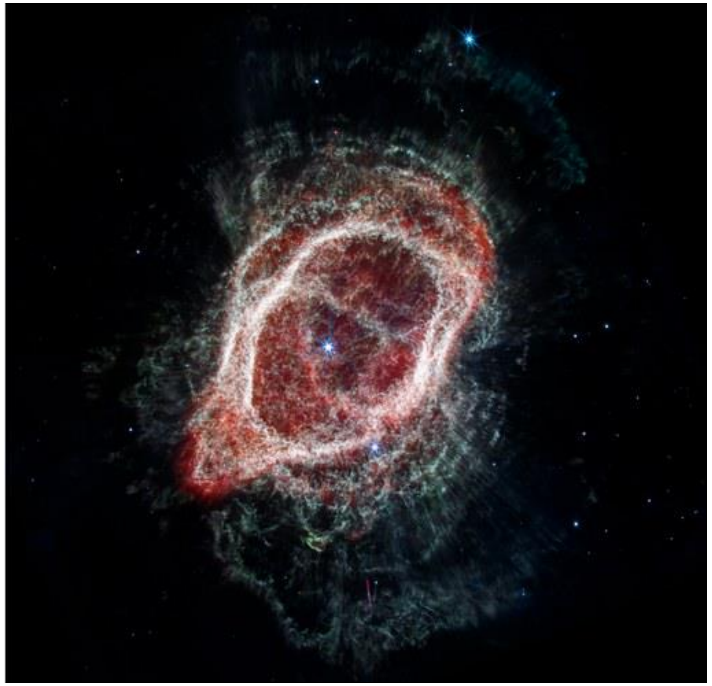
So, the electrons need energy to escape that potential.

Electron can be freed if it has enough energy to escape the potential well.

This typically requires an energy of a few electron volts (eV). For Hydrogen the ionisation energy is 13.6eV (from the ground state). Note 1eV (energy) ~ 11,000 Kelvin (temperature)

Plasma in the Universe

Plasmas are the most common phase of matter. Over 99% (or 95%, 99.9%) of the visible universe is known to be plasma

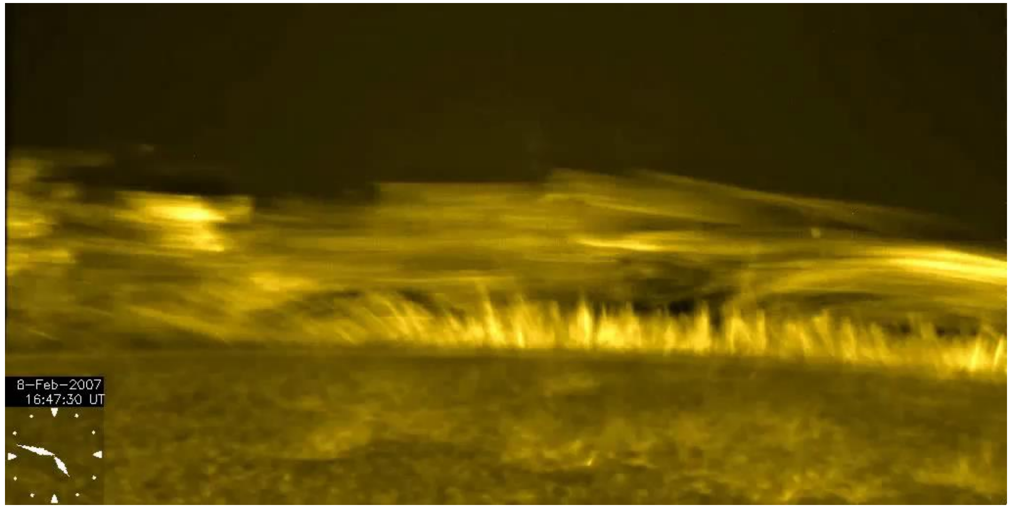


Southern Ring Nebula
JWST (NASA)

The Pillars of Creation (Webb NIRCam Image) - image courtesy of NASA

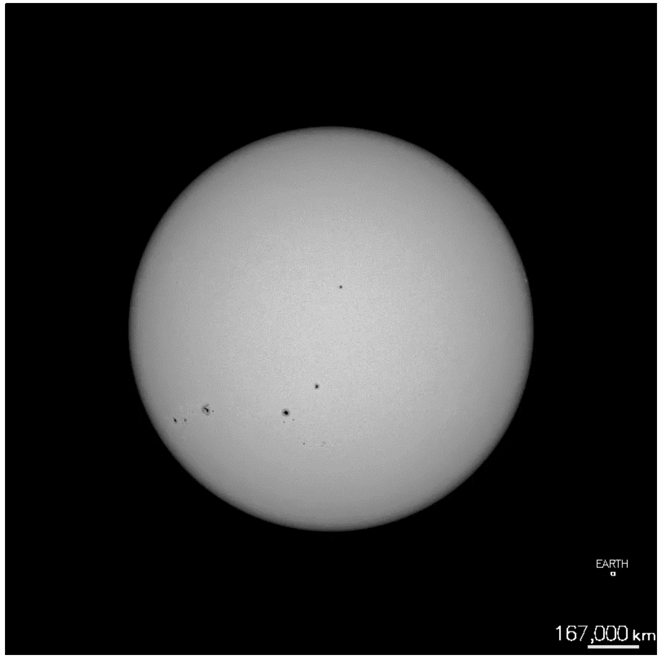
Galaxies in the distant universe
imaged by JWST Deep Field (NASA)

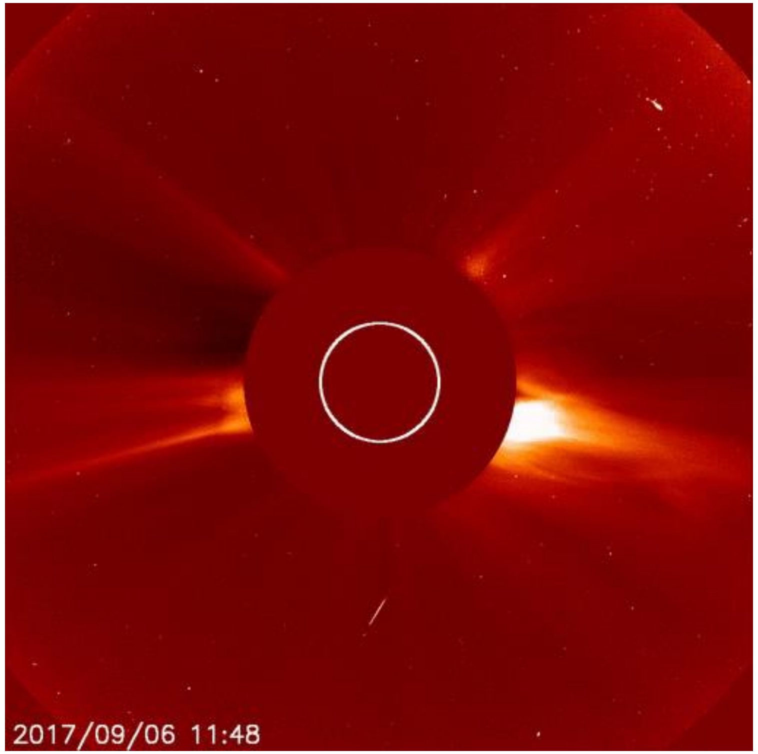
Plasma in the Solar System



Chromosphere (Hinode)

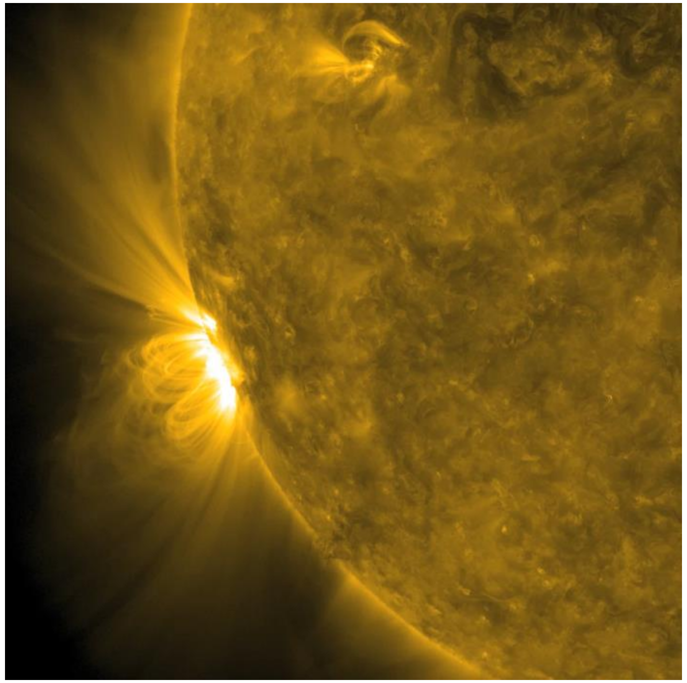
Photosphere (Hinode)



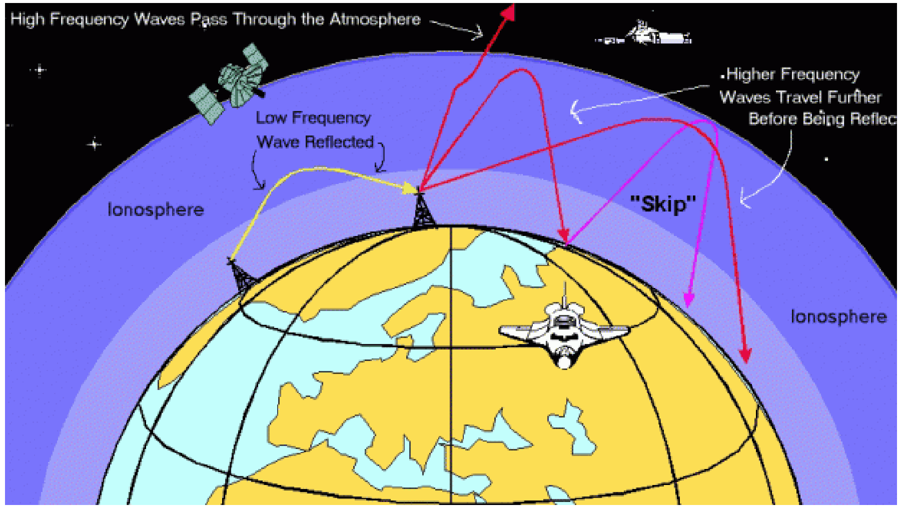


Solar Wind

Corona (AIA/SDO)

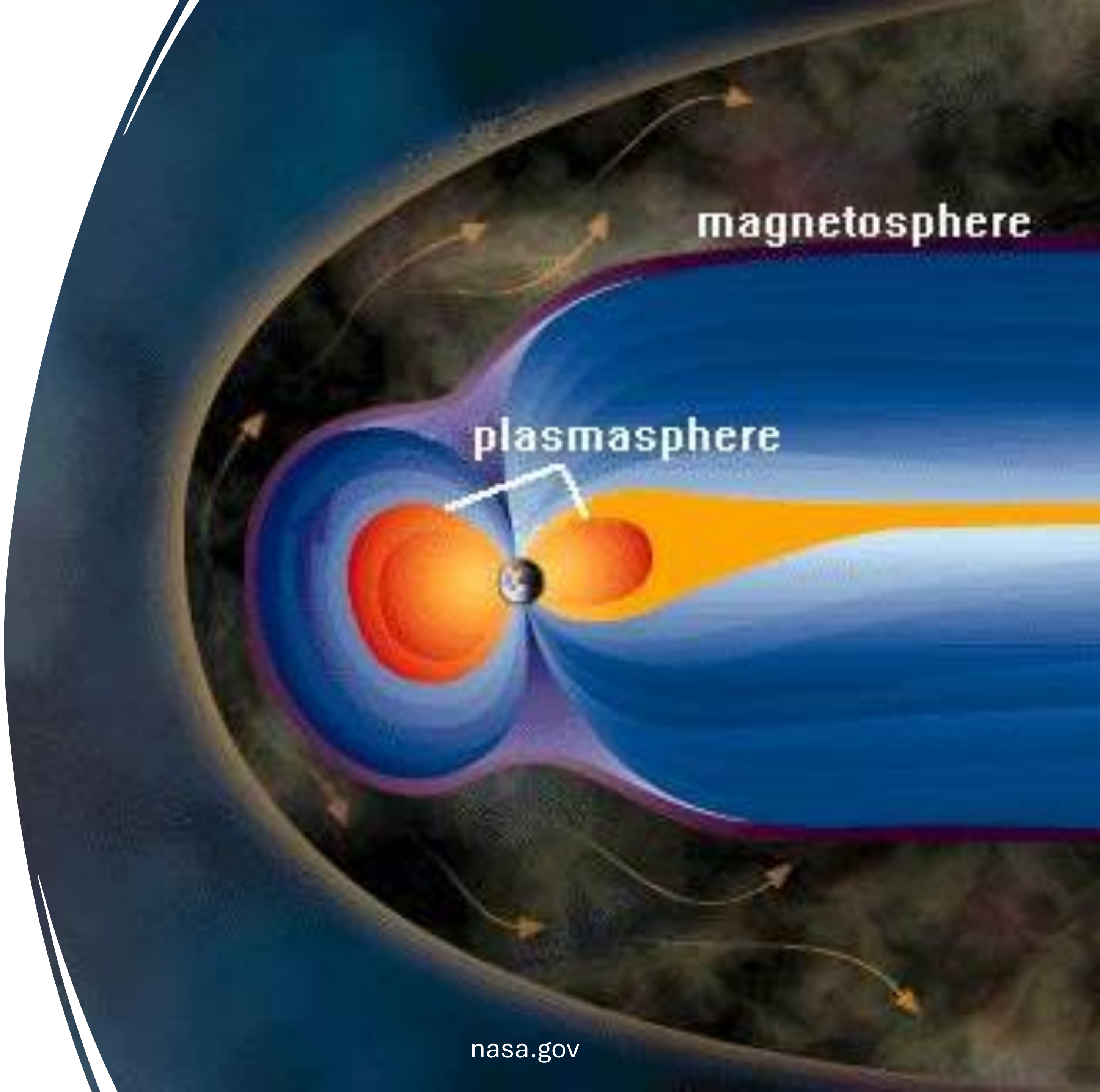


Plasma around the Earth



Ionosphere

swpc.noaa.gov



nasa.gov

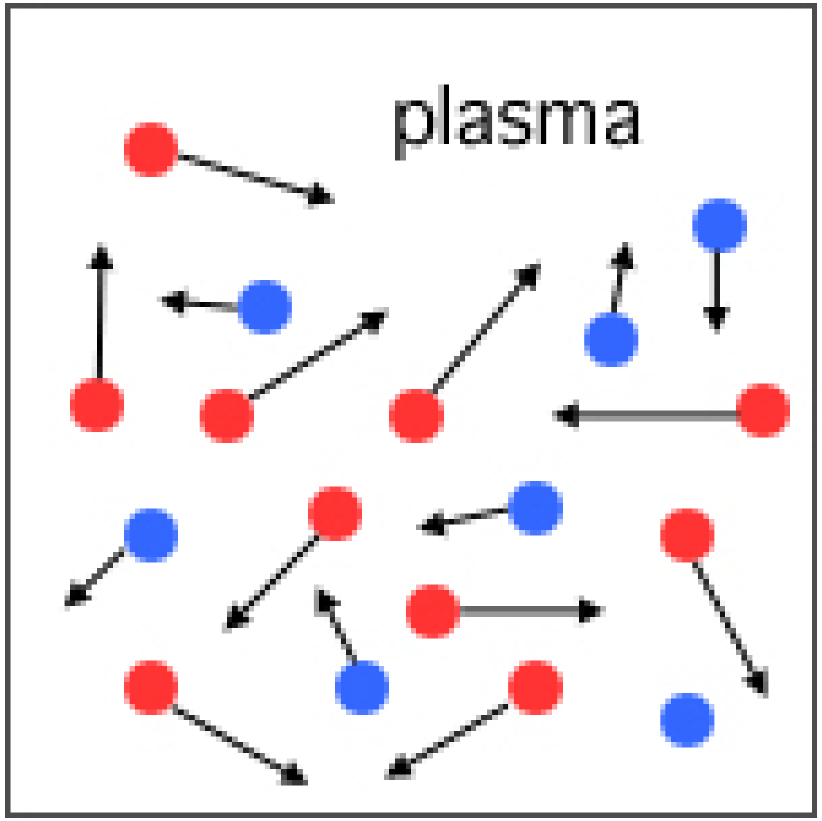
Temperature and how it is defined

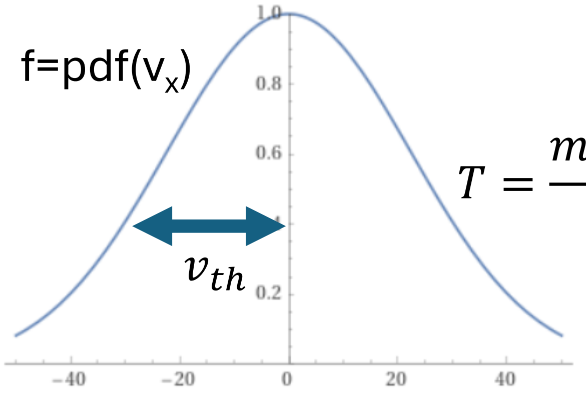
Temperature relates to the random motions of particles (with flow relating to the average motion).

The thermal velocity of particles is related to the standard deviation of the particle velocity

$$v_{th} \propto \sqrt{T}$$

Temperature does not have to be isotropic





$$T = \frac{m_p {v_{th}}^2}{k_b}$$

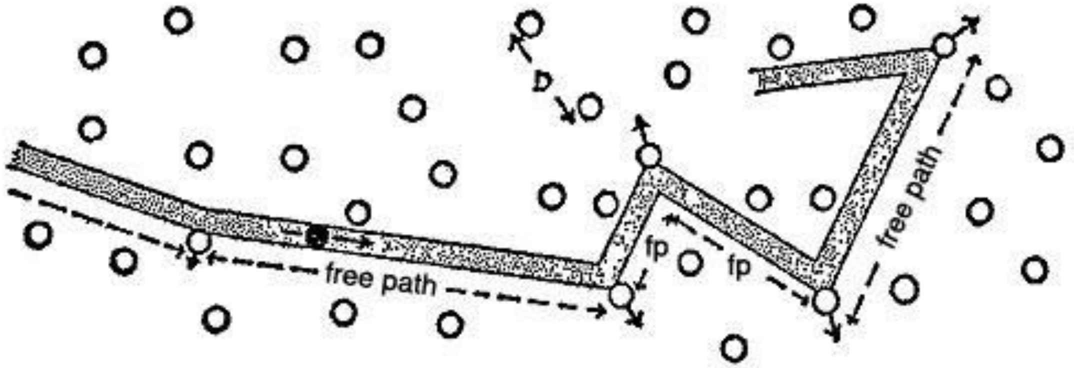
Key plasma quantities – Particle collisions

Given enough time (or enough particles) there will be particle-particle collisions.

In plasma these are Coulomb collisions (based on electric field interactions). The frequency is given by

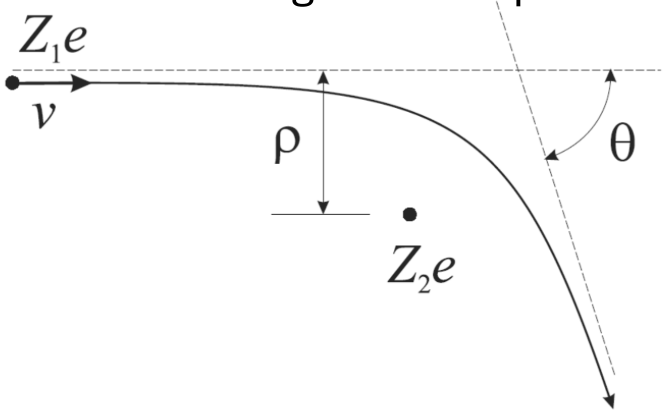
$$\nu_{\mathrm{ei}} = \frac{\pi^{3/2} n_{\mathrm{e}} Z e^4 \ln \Lambda}{2^{1/2} (4\pi\varepsilon_0)^2 m_{\mathrm{e}}^2 v_{\mathrm{te}}^3} \ \mathrm{s}^{-1},$$

This includes the Coulomb logarithm which typically has a value between 10 and 20



Mean free path of a gas molecule

Image from http://sun.stanford.edu/



$$\ln \Lambda = \ln \frac{r_D m v^2}{2 Z_1 Z_2 e^2} = \ln \frac{3T^{3/2}}{2(4\pi)^{1/2} Z_1 Z_2^2 e^3 n^{1/2}}.$$

For example, see http://sun.stanford.edu/ or https://scipub.euro-fusion.org/, https://farside.ph.utexas.edu/

Maxwellian velocity distributions

With enough collisions the temperature of a plasma is well defined with the distribution of random velocities becoming a Maxwellian.

Generally, the core of the distribution relaxes faster to the Maxwellian than the tails of the distribution.

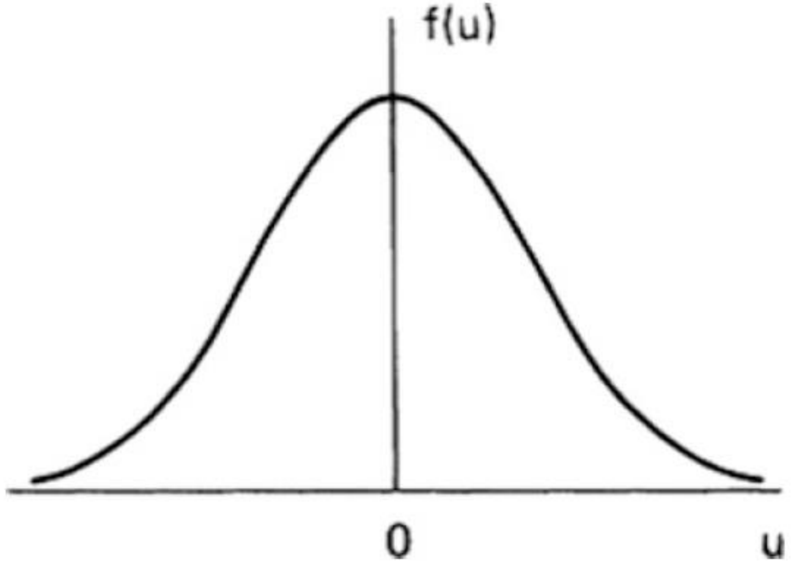


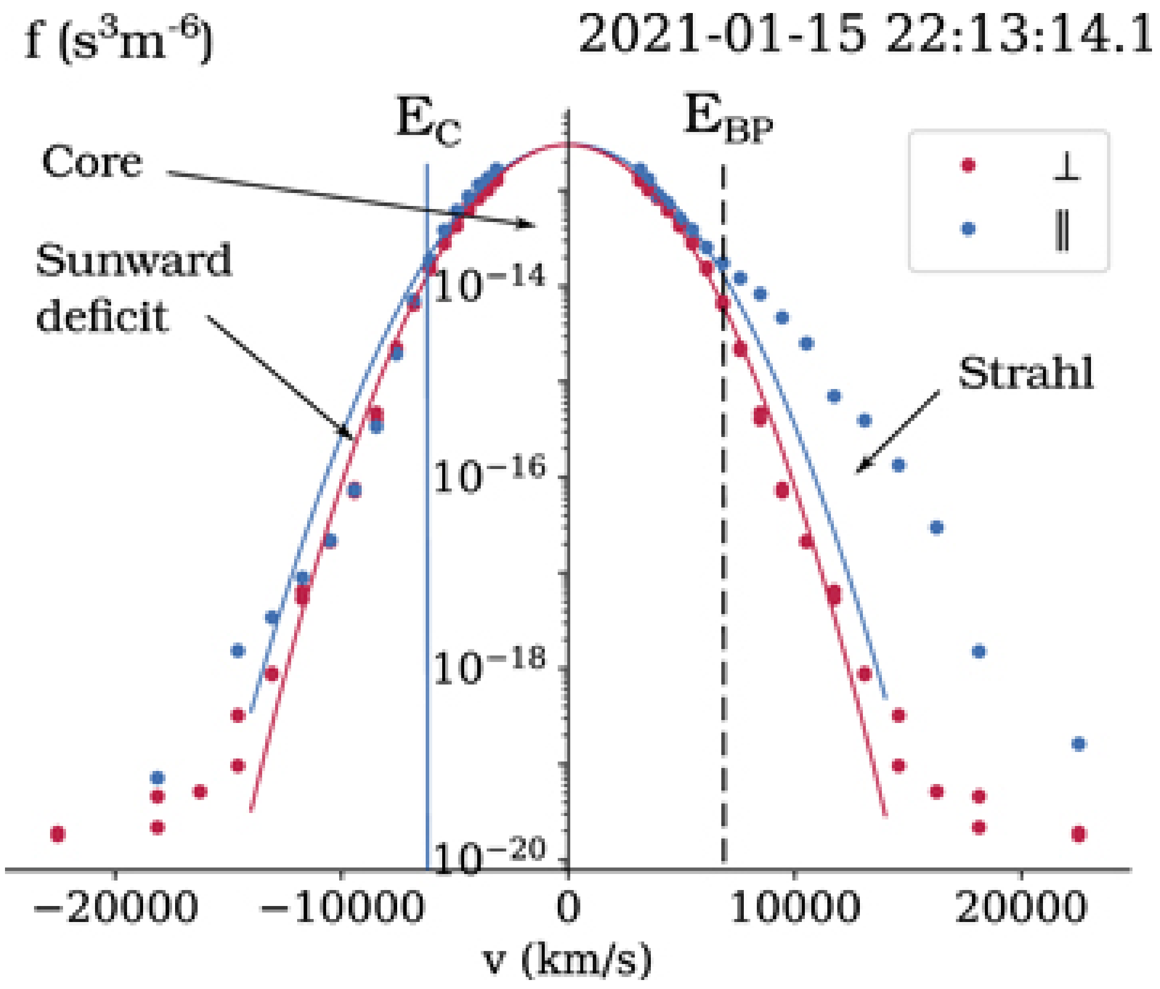
Fig. 1.2 A Maxwellian velocity distribution

$$f_s(\boldsymbol{x}, \boldsymbol{v}, t) = \frac{n_s(\boldsymbol{x}, t)}{(\sqrt{2\pi} v_{th,s})^3} e^{-(\boldsymbol{v} - \boldsymbol{V}_s(\boldsymbol{x}, t))^2/(2v_{th,s}^2)}$$

Quasi-collisionality in the young solar wind

On the right I show an example of the velocity distribution in the young solar wind.

We can see that a lot of velocity distribution follows Maxwellians (solid lines) but the temperature is anisotropic, and we have non-Maxwellian components of the distribution.



Laura Berčič *et al* 2021 *ApJ* **921** 83

Collisions with non-charged particles

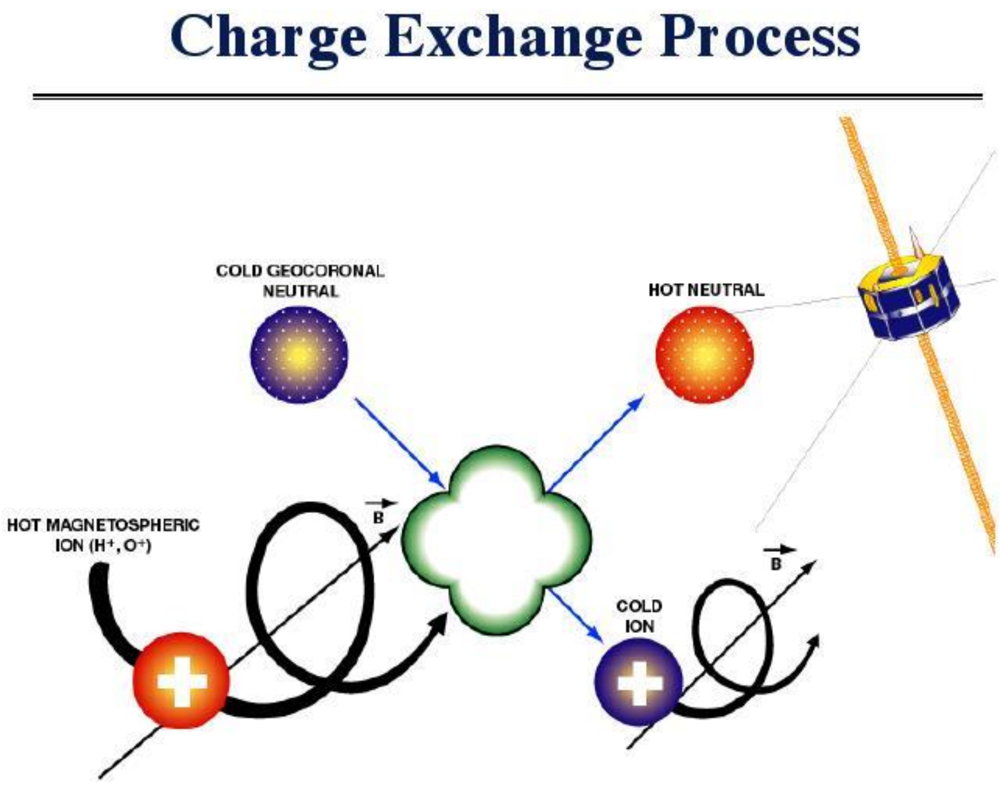
In a plasma, often not all particles are charged (so do not undergo Coulomb collisions). However, we cannot get collective behaviour without particles interacting.

Beyond ionisation and recombination, there are two key interactions: hard-sphere collisions and charge exchange.

To good approximation frequency is given by

$$\nu_{in} = n_n \sqrt{\frac{8k_B T}{\pi m_{in}}} \sigma_{in} ,$$

Image from https://image.gsfc.nasa.gov/



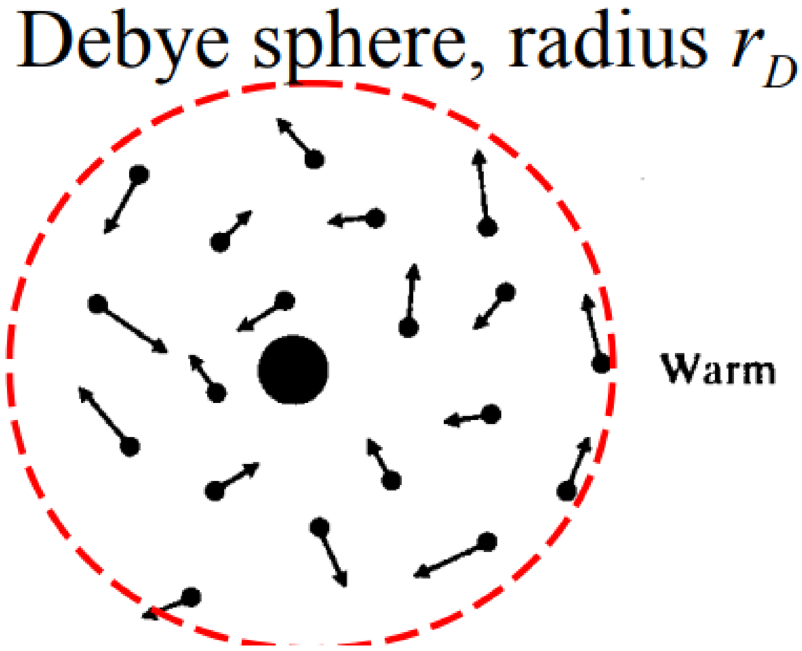
Key plasma quantities – Debye Shielding

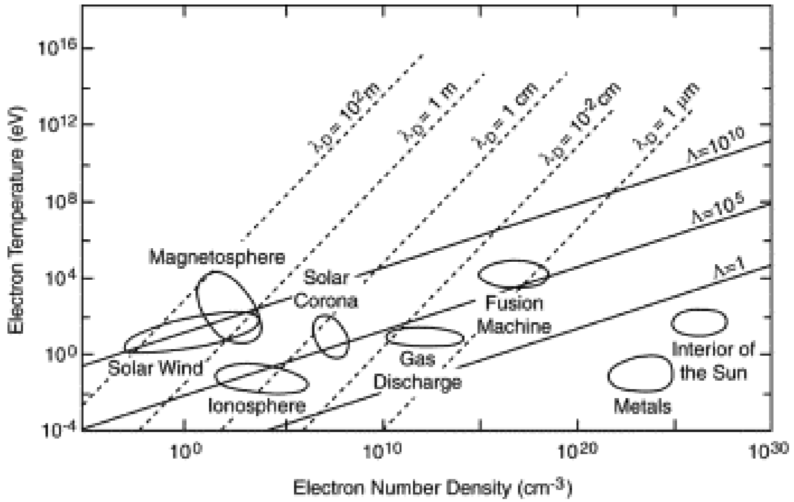
Plasmas are made up of charged particles, but when looked at on a large enough scale they appear to be charge neutral.

This is because like charges repel and opposite charges attract (and electrons move very freely).

This means above a certain distance from a charge (the Debye radius) the electric field of the plasma is approximately zero

$$\lambda_D \equiv \left(\frac{\varepsilon_0 K T_e}{n e^2} \right)^{1/2}$$





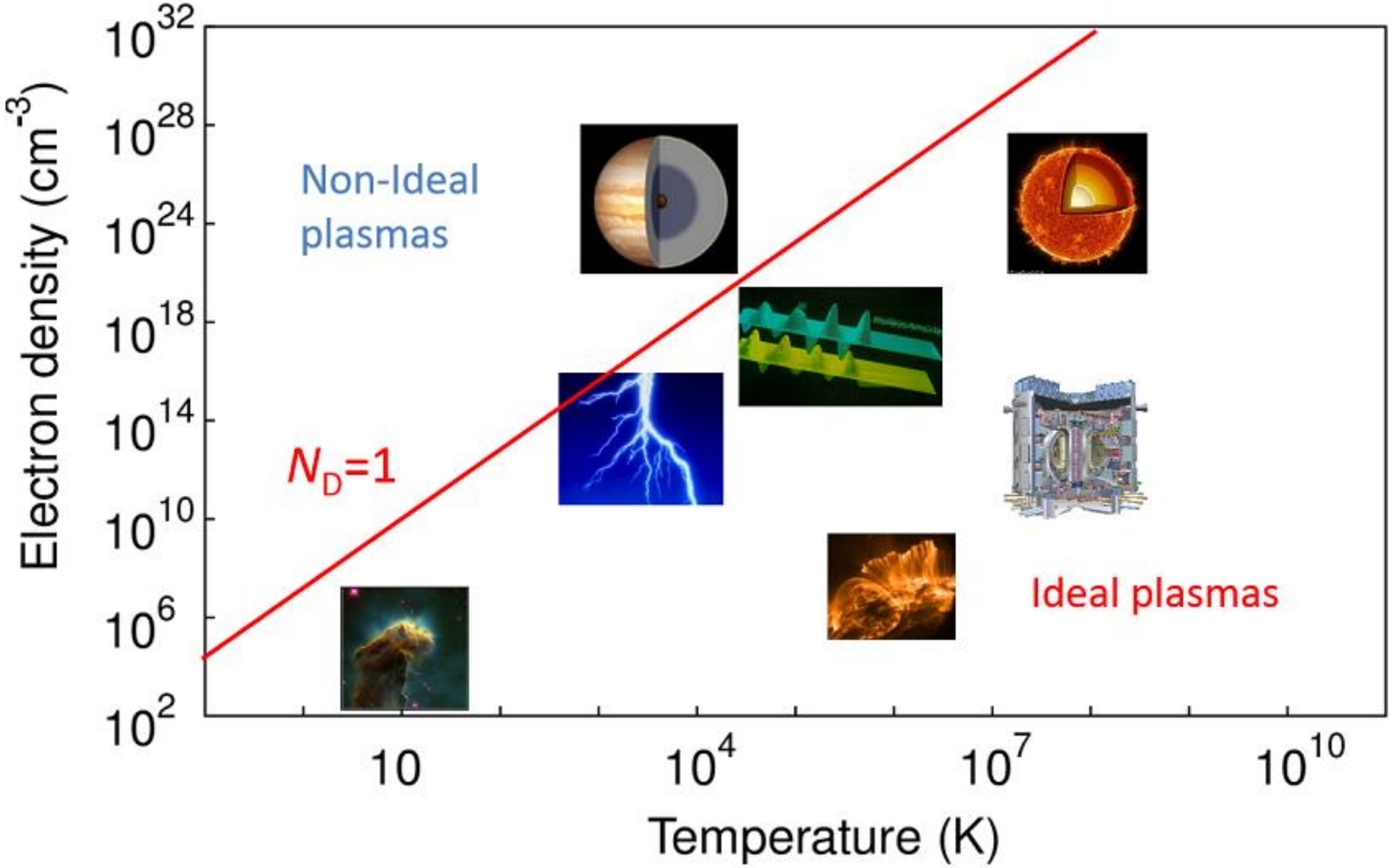
C.T. Russell, in Encyclopedia of Physical Science and Technology (Third Edition), 2003

Collective behaviour (ideal plasmas)

To get collective behaviour in the plasma, you need to have a lot of particles per Debye sphere.

Generally, this is described by the following criterion

$$N_D \equiv n_e \frac{4\pi}{3} \lambda_D^3 \gg 1,$$



Introduction to Plasma Physics / Gibbon, P. (Julich, Forschungszentrum ; IAS, Julich)

Maxwell's equations

$$J = \sum_s q_s n_s \boldsymbol{v}_s$$

B – Magnetic Field
E – Electric Field
J – Current density
σ – Charge density $\quad \sigma = \sum_s q_s n_s$

Faraday's law
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t,$$

Ampere's law
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t},$$

Gauss's law
$$\nabla \cdot \mathbf{E} = \sigma/\varepsilon_0,$$

Solenoidal Constraint
$$\nabla \cdot \mathbf{B} = 0.$$

So, what do they all mean?

- Time varying magnetic fields create Electric fields
- Currents (including the displacement current) create magnetic fields
- Charges create electric fields
- There are no sources or sinks for magnetic fields.

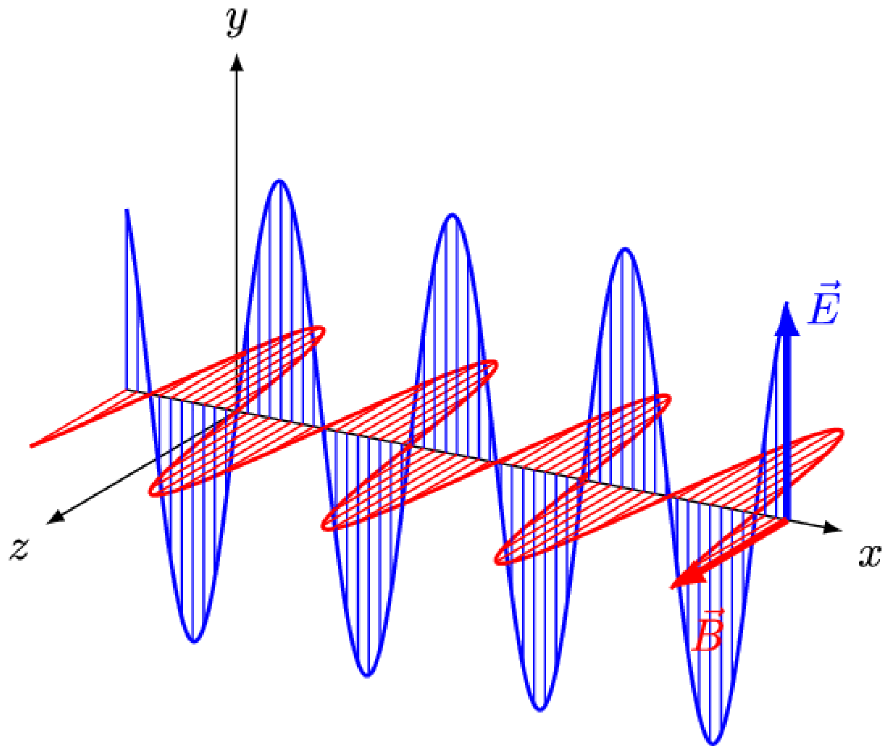
Waves in a vacuum (a little reminder of UG Electromagnetism)

In a vacuum, Faraday's and Ampere's law are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

These can be combined to give a wave equation (here we take propagation in the x direction)

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$



2 points: B perturbation orthogonal to E perturbation

These waves travel at the speed of light

The Lorentz Force and particle motions

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = \frac{d(\gamma m_0 \boldsymbol{v})}{dt} = \mathrm{q}\,(\mathbf{E} + \mathbf{v} \times \boldsymbol{B})$$

m_0 rest mass ; $\gamma = 1/(1 - v^2/c^2)$ relativistic gamma ; q the signed particle charge

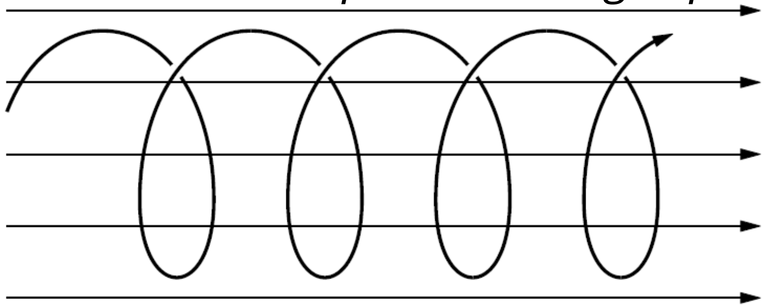
The case of constant magnetic field - *E* = *0*, *but **B** = (0,0,B_0)*

$$\boldsymbol{F} = \mathrm{q}\,(\mathbf{v} \times \boldsymbol{B})$$

$$\frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{v}}{dt} = 0$$

Magnetic fields do no work i.e. cannot change the energy of a particle

For a positive charged particle



Pitch angle α is the angle between particles velocity vector and the local magnetic field

The frequency with which a particle gyrates around the magnetic field is called the Gyro-/Cyclotron-/Larmor-frequency

$$\omega_c = q\,|\boldsymbol{B}|/(m_0\gamma)$$

Note that positive and negative particles gyrate in opposite directions

Gyro-/Cyclotron-/Larmor-radius $r_L = v_\perp / |\omega_c|$ (this gets smaller for large B or small mass)

Particle drifts and the adiabatic invariants

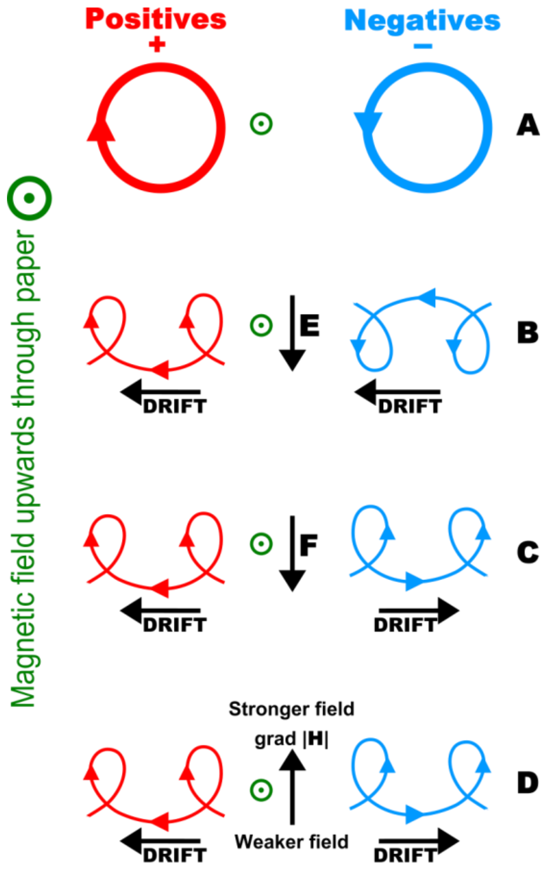


Image from https://en.wikipedia.org/

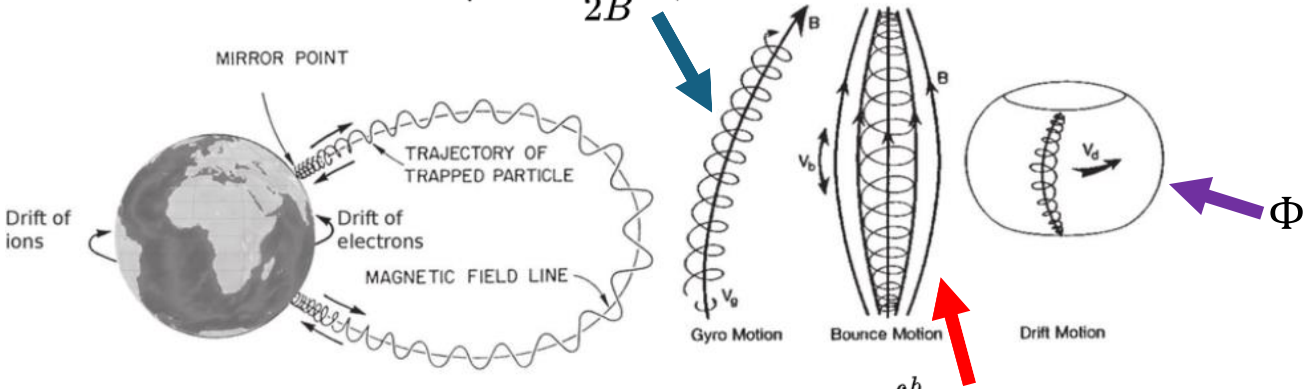
If Larmor radius is small compared to the system gradients then can superpose drifts on to the normal gyromotion.

The component of a force perpendicular to the magnetic field drives a velocity drift of the form

$$\boldsymbol{v}_f = \frac{1}{q} \frac{\boldsymbol{F} \times \boldsymbol{B}}{B^2}.$$

Adiabatic invariants

$$\mu = \frac{\gamma m_0 v_\perp^2}{2B},$$



$$J = \int_a^b p_\parallel \, ds,$$

See: Northrop, T G (1961). "The guiding center approximation to charged particle motion". *Annals of Physics*. **15** (1): 79–101.

The Lorentz Force and particle motions

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = \frac{d(\gamma m_0 \boldsymbol{v})}{dt} = \mathrm{q}\,(\mathbf{E} + \mathbf{v} \times \boldsymbol{B})$$

m_0 rest mass ; $\gamma = 1/(1 - v^2/c^2)$ relativistic gamma ; q the signed particle charge

Constant electric field - *B* = *0*, *but **E** = (0,0,E_0)*

$$\boldsymbol{F} = \mathrm{q}\mathbf{E}$$

$$\frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{v}}{dt} = \frac{qE_0}{\gamma m_0}$$

A constant electric field accelerates particles indefinitely

It will almost but never quite reach c

Key plasma quantities – Langmuir waves

A fundamental aspect of plasmas are the plasma oscillations

Let us consider of quasi-neutral plasma in which ions are static and electrons freely moving. Let us look at a 1D case of an electron that has been displaced by distance x. The Poisson equation for electric field,

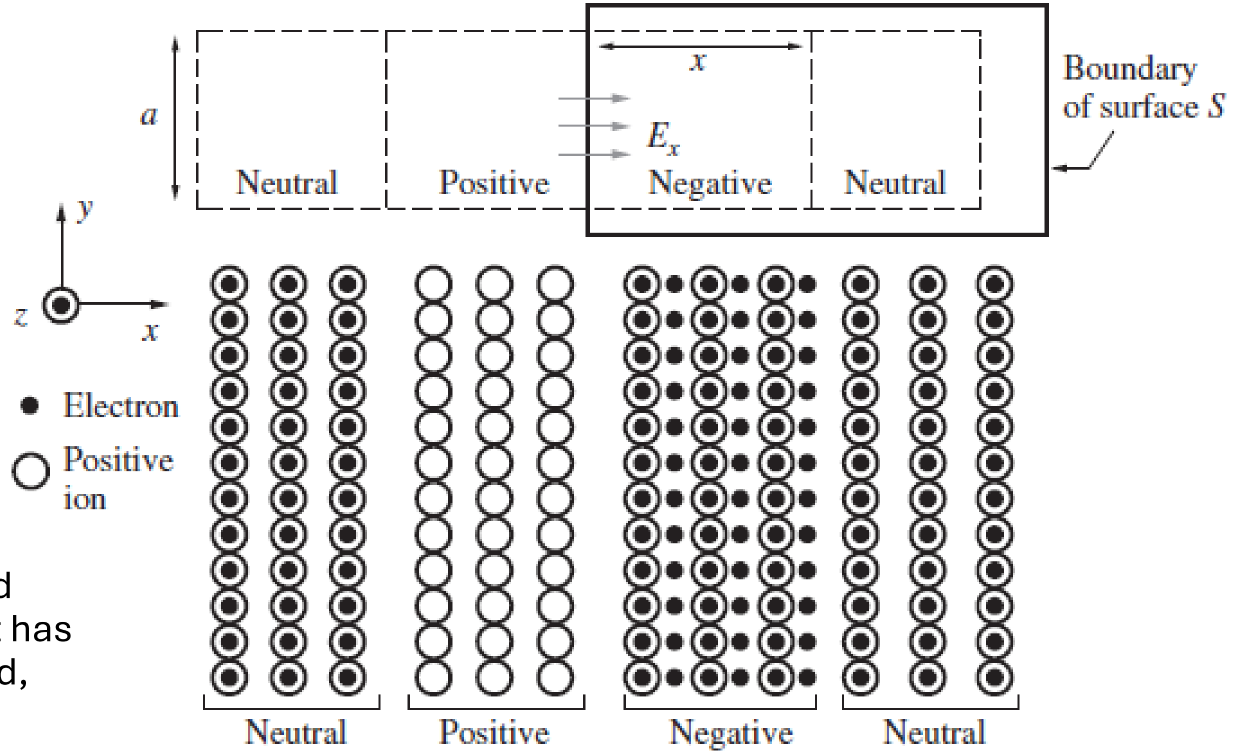
$$\nabla \cdot \mathbf{E} = \sigma/\varepsilon_0, \qquad \frac{\partial E}{\partial x} \approx \frac{E}{x} = \frac{e\, n_0}{\varepsilon_0} \Rightarrow E = \frac{e\, n_0}{\varepsilon_0} x$$

and Newton's 2nd law give:

$$m_e \vec{a} = \vec{F} \Rightarrow m_e \ddot{x} = -eE = \frac{e^2 n_0}{\varepsilon_0} x \Rightarrow \ddot{x} = -\frac{e^2 n_0}{\varepsilon_0 m_e} x \equiv -\omega_{pe}^2 x$$

This gives the plasma frequency

$$\omega_{pe} = \sqrt{\frac{e^2 n_0}{\varepsilon_0 m_e}} \approx 56.4\sqrt{n_0[m^{-3}]}$$



As an example, for the solar corona we have $\omega_{pe} \approx 10^9$ s^{-1} rad

Waves in plasmas – Dielectric tensor

Let's start back at Faraday's and Ampere's law (but this time not in a vacuum)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

By assuming a uniform, infinite plasma with no electric field and uniform magnetic field, to understand the linear waves we can look for normal modes of the form

$$\exp i(\mathbf{k}.\mathbf{x} - \omega t)$$

Then by combining and manipulating we can show

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2}\mathbf{E} + i\omega\mu_0 \mathbf{J} = 0$$

We need an equation to connect **E** and **J** to close this system, the most general linear relationship

$\mathbf{J} = \overleftrightarrow{\boldsymbol{\sigma}} \cdot \mathbf{E}$ with $\overleftrightarrow{\boldsymbol{\sigma}}$ called the conductivity tensor

Waves in plasmas – Dielectric tensor

Substituting this in gives

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2}\mathbf{E} + i\omega\mu_0 \overleftrightarrow{\boldsymbol{\sigma}} \cdot \mathbf{E} = 0$$

We can rewrite this using the dielectric tensor

$$\overleftrightarrow{\boldsymbol{\epsilon}} = \overleftrightarrow{\mathbf{I}} + \frac{i}{\omega \varepsilon_0}\overleftrightarrow{\boldsymbol{\sigma}}$$

giving

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2}\overleftrightarrow{\boldsymbol{\epsilon}} \cdot \mathbf{E} = 0$$

This can be written as

$$\overleftrightarrow{\mathbf{D}} \cdot \mathbf{E} = 0 \text{ with } \overleftrightarrow{\mathbf{D}} = \mathbf{k}\mathbf{k}^{\mathrm{T}} - k^2\overleftrightarrow{\mathbf{I}} + \frac{\omega^2}{c^2}\overleftrightarrow{\boldsymbol{\epsilon}}$$

and our non-trivial solutions exist when $\det\left|\overleftrightarrow{\mathbf{D}}\right| = \mathbf{0}$

Good derivation given on https://ocw.mit.edu/

The Zoo of Waves in Plasmas - I

Looking at the longitudinal oscillations, we can see key physics emerging from the associated plasma frequencies.

For example, note the behaviour of the ion acoustic wave as it approaches the ion plasma frequency

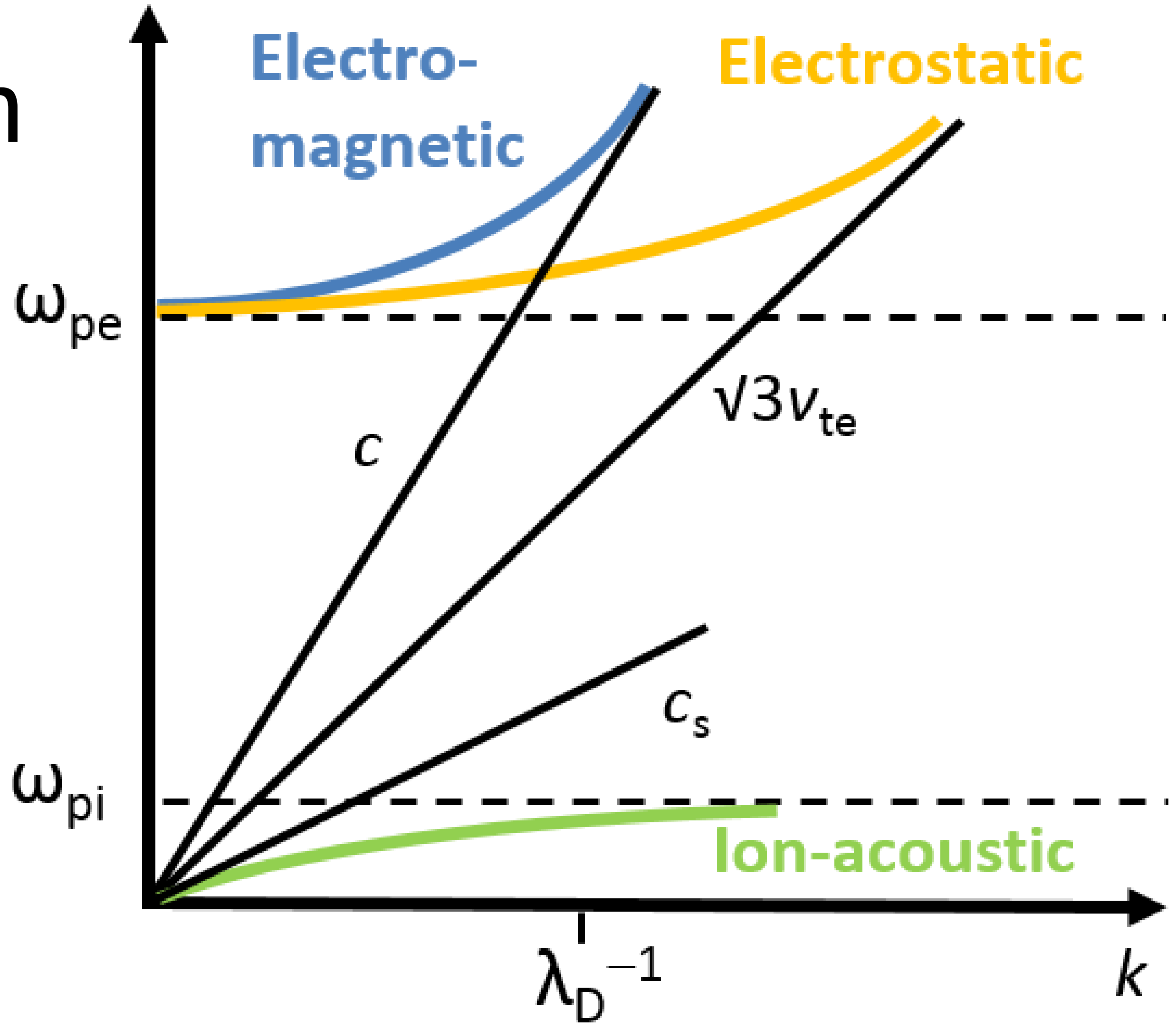
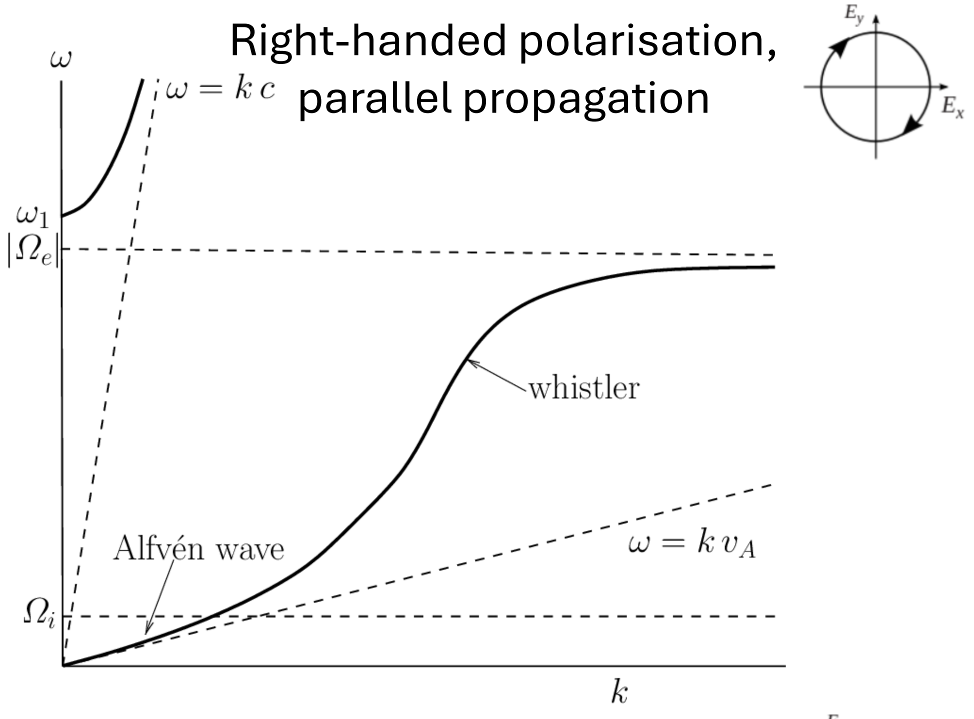


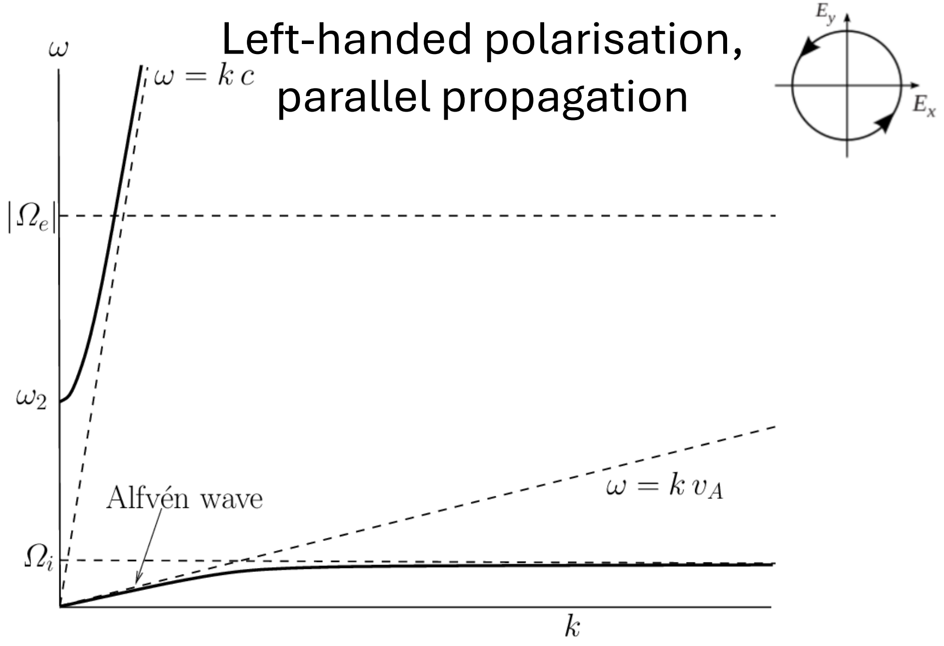
Image from https://cds.cern.ch/

The Zoo of Waves in Plasmas - II

Now we look at circular polarised waves (for field-aligned propagation). Of note is what happens to the Alfven wave, the left-hand polarised wave phase speed saturates at the ion gyrofrequency (as it has the same polarisation as the ions, but the right-hand polarised wave becomes the whistler wave (associated with electrons).

You will hear more about waves (probably at MHD scales) later today.



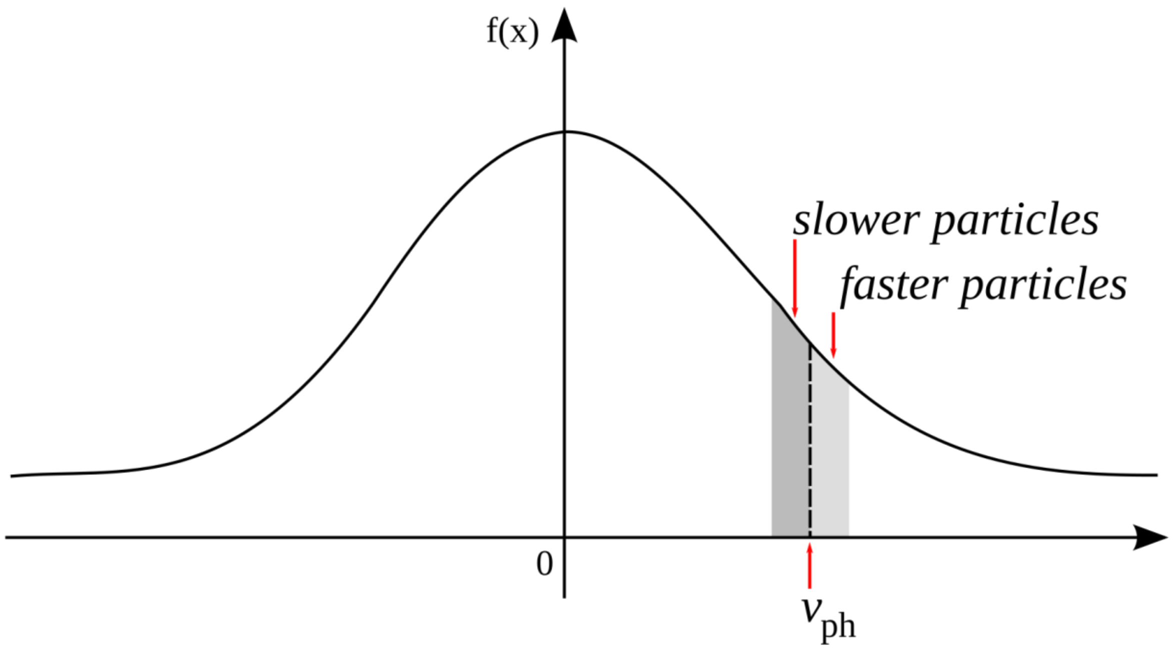


https://farside.ph.utexas.edu/

Wave-particle interaction

With many characteristic frequencies of a plasma (e.g. gyro-frequency, plasma frequency) there is the great potential that waves and particles can interact with energy flowing from one to the other.

One important example of this is Landau damping. This works for longitudinal waves and acts where the phase speed of the wave approximately matches the particle velocity in phase space.



Particles travelling a little slower than the wave extract energy from the wave, but those travelling a little faster give energy to the wave. For a Maxwellian, more particles travel slower so the wave damps.

But not all particle distributions are Maxwellian.

Modelling plasma motion – distribution functions

For a species α we can define the distribution function $f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$ such that

$$f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{v} = \mathrm{d}N(\boldsymbol{r}, \boldsymbol{v}, t)$$

is the number of particles in the volume $\mathrm{d}V = \mathrm{d}\boldsymbol{v}\, \mathrm{d}\boldsymbol{r}$

Number density: $n_\alpha = \int f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{d}\boldsymbol{v}$

First velocity moment: $\overline{\boldsymbol{v}(\boldsymbol{r}, t)} = \dfrac{1}{n(\boldsymbol{r}, t)} \int \mathrm{d}\boldsymbol{v}\, \boldsymbol{v}\, f(\boldsymbol{r}, \boldsymbol{v}, t)$

Second velocity moment: $P_{ij} = m \int \mathrm{d}\boldsymbol{v}\, v_{r\,i} v_{r\,j}\, f(\boldsymbol{r}, \boldsymbol{v}, t)$
or pressure tensor

In thermal equilibrium, the distribution of velocities becomes the Maxwellian distribution we discussed earlier

Some nice lecture notes here (anu.edu.au)

Modelling plasma motion – Boltzmann Equation

We can describe the temporal evolution of the distribution function by

$$\frac{\partial f}{\partial t} = -\boldsymbol{\nabla}_r.(\boldsymbol{v}f) - \boldsymbol{\nabla}_v.(\boldsymbol{a}f) + \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

Effect of collisions

For plasmas, the dominant force is electromagnetic, i.e. the Lorentz force

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) \equiv m\boldsymbol{a}.$$

This leads to the Boltzmann equation (when the collision term is zero this is called the Vlasov Equation)

$$\frac{\partial f}{\partial t} + \boldsymbol{v}.\nabla_r f + \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}).\nabla_v f = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

Modelling plasma motion – towards a fluid description

In your next lecture you are going to learn about MHD. The plasma description (including the distribution function, moments and their evolution) connects directly to this given you are studying dynamics that are slow enough (compared to gyro-frequency, collision frequency) and large enough (compared to Debye length, mean-free path) along with some other assumptions.

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{V}) = 0, \qquad \text{Mass Continuity Eq.,}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\frac{P}{\rho^{\gamma}}\right) = 0, \qquad \text{Energy Eq.,}$$

$$\rho \frac{\mathrm{d}\,\mathbf{V}}{\mathrm{d}\,t} = -\nabla P - \frac{1}{\mu_0}\mathbf{B} \times (\nabla \times \mathbf{B}), \qquad \text{Euler's Eq.,}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \qquad \text{Induction Eq..}$$

Summary

In this lecture we have

- Heard what a plasma is and the fact almost everything we see in the Universe is plasma
- What are the key aspects (charge neutrality, Coulomb collisions, Maxwellian distributions)
- What electric and magnetic fields do to plasmas, including the waves that we find in plasmas
- Looked at techniques to model plasmas and connection to MHD

Textbooks I use a lot (though this shows my bias towards MHD)